The No Alternatives Argument

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Motivation

- Question: How are scientific theories assessed? What is a good scientific theory?
- Traditional answer: Theories are assessed in the light of empirical data.
- However: This does not work too well in fundamental physics. In some cases one has to wait very long for an empirical confirmation (e.g. in the case of the Higgs), and in other cases it is not clear whether there will ever be empirical data (e.g. in the case of superstring theory).
- What can be done? Are there "non-empirical" ways of assessing scientific theories?
- Some people believe so, and I will assess the corresponding argument structure.

Overview

II. Theory Assessment and the NAA

Theory Assessment I: The HD-Model

According to the hypothetico-deductive model a theory or hypothesis H is confirmed by a piece of evidence E iff E is predicted by H (i.e. if E is a deductive consequence of H) and if E is observed.

Some remarks:

1. Note that a theory can only be confirmed empirically.
2. The hypothetico-deductive model has a number of problems, e.g.
   - The Tacking Problem: If E confirms H, then it also confirms $H \wedge X$.
   - Confirmation is a yes-no matter: there are no degrees of confirmation.
   - Etc.

Theory Assessment II: Bayesian Confirmation Theory

According to the Bayesian Confirmation Theory a theory or hypothesis H is confirmed by a piece of evidence E iff the observation of E raises the probability of H. Here is an illustration.

- A Bayesian agent entertains the following hypothesis:
- H: It will rain tomorrow in Aachen.
- She assigns a prior probability of, say, $P(\mathrm{H}) = .4$ to it.
- Next, she listens to the radio and hears the weatherman saying that it will rain tomorrow in Cologne. Hence, she takes as evidence for H the following proposition:
- E: The weatherman says that it will rain tomorrow in Cologne.
- She then updates her beliefs according to Bayes Theorem: $P'(\mathrm{H}) = P(\mathrm{H}|\mathrm{E})$. Here $P'(\mathrm{H})$ is the posterior probability of H.
- We might then obtain, say, $P'(\mathrm{H}) = .6$: E confirms H.

Theory Assessment II: Bayesian Confirmation Theory

To calculate the posterior probability, it is useful to recall the mathematical identity ("Bayes Rule"):

$$P(\mathrm{H}|\mathrm{E}) = \frac{P(\mathrm{E}|\mathrm{H}) \cdot P(\mathrm{H})}{P(\mathrm{E})}.$$

Note:

- If E is a deductive consequence of H, then $P(\mathrm{E}|\mathrm{H}) = 1$ and $P'(\mathrm{H}) = P(\mathrm{H}|\mathrm{E}) = P(\mathrm{H})/P(\mathrm{E})$, i.e. more surprising evidence (i.e. less expected evidence) confirms better than less surprising evidence.
- One can account for the truism that theories can be more or less well confirmed.
- The probabilities in question are based on the subjective judgment of scientists or the scientific community.

Discussion

- Bayesian Confirmation Theory can be applied in a straightforward way to empirical testing.
- However, there are other ways of confirming or supporting a theory. Examples: (i) a theory explains a novel phenomenon for which it was not constructed. (ii) a theory coheres well with accepted theories. (iii) a theory belongs to a class of theories that have been successful (e.g. it is a gauge theory).
- We ask: Can they be accounted for in Bayesian Confirmation Theory?
- Another example: The No Alternatives Argument.

The No Alternatives Argument (NAA)

Scientists often argue like this:

1. Hypothesis H satisfies several desirable conditions (incorporates various principles, coheres with other theories, ...)
2. Despite a lot of effort, the scientific community has not yet found an alternative to H.
3. Hence, we have one reason in support of H.

We ask:

- How good are NAAs?
- Under what conditions do they work?

Examples from Fundamental Physics

There are many examples of NAAs in fundamental physics, mainly because discriminating empirical evidence is hard to come by. Here are three:

1. The Higgs Mechanism
   This mechanism was invented in 1964. Its empirical confirmation had to wait till 2012. However, there was little doubt in the scientific community that the model was in principle correct. There was no alternative...
2. String Theory
   This theory cannot (yet) be tested empirically. What speaks in its favor are (mostly unproven) coherence arguments and the NAA.
3. Cosmic Inflation
   This theory enjoys a very limited degree of empirical confirmation. Trust in the theory crucially relies on the NAA.
   This is a nice example which can be used to study how empirical and non-empirical (NAA) confirmation can work together.

III. A Bayesian Analysis

Formalizing NAAs

- Let $\mathcal{D}$ be a set of data and $\mathcal{C}$ be a set of constraints. The community aims at a theory that accounts for $\mathcal{D}$ & satisfies $\mathcal{C}$.
- The hypothesis H accounts for $\mathcal{D}$ and satisfies $\mathcal{C}$.
- So far, no alternative hypothesis has been found that explains $\mathcal{D}$ and satisfies $\mathcal{C}$.
- We ask: To what extend does this observation confirm H?

We introduce two propositional variables:

1. T has two values, viz. T: The hypothesis H is empirically adequate, and $\neg$T: The hypothesis H is not empirically adequate.
2. F also has two values, viz. F: The scientific community has not yet found an alternative to H that accounts for $\mathcal{D}$ and fulfills $\mathcal{C}$, and $\neg$F: The scientific community has found an alternative to H that explains $\mathcal{D}$ and fulfills $\mathcal{C}$.

A First Bayesian Attempt

Goal: Show that F confirms T, i.e. that

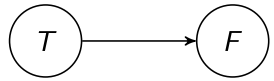
$$d(\mathrm{T},\mathrm{F}) := P(\mathrm{T}|\mathrm{F}) - P(\mathrm{T}) > 0.$$

$d(\mathrm{T},\mathrm{F})$ is the difference measure of confirmation. $P(\mathrm{T})$ is the prior probability of T, and $P(\mathrm{T}|\mathrm{F})$ is the posterior probability of T, i.e. the probability of T after having learned the evidence F.

- How can this be done?
- Wave your hands, and claim that F is *obviously* positively relevant for T.
- But why should we accept this? And even if we do so, the Bayesian machinery does not add much or anything here.

Another Worry

Here is a standard Bayesian Network representation that is used to model the relation between a hypothesis and a piece of evidence:



- On this account, there is a direct influence of the hypothesis on the evidence. (Example: T: All raven are black. F: This raven is black.)
- But no such relation holds in the present case. F is at best some kind of indirect evidence for T. F is an example of what we call non-empirical evidence.
- Hence our conjecture: There is a common cause variable Y that mediates the correlation between F and T. Technically speaking, Y screens off F and T.
  Illustration: yellow fingers ← smoking → heart disease

Introducing Y

We introduce a third variable.

3. Y has N values, viz. Y_i: There are exactly i hypotheses which explain $\mathcal{D}$ and fulfill $\mathcal{C}$. (H is one of them.)

Note that N can be ∞.

- The alternative theories make different predictions, and can therefore be individuated. Theories that make exactly the same (or almost the same) empirical predictions, are considered to be identical. Scientists have a good sense of what counts as a different theory, and what not.
- We claim that scientists have beliefs (supported by arguments) about the distribution of the Y_i.

Relations between F, T and Y

Repetition:

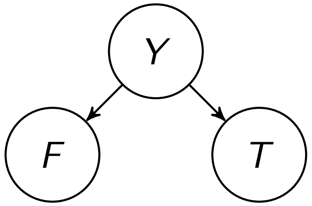
1. T has two values, viz. T: The hypothesis H is empirically adequate, and ¬T: The hypothesis H is not empirically adequate.
2. F also has two values, viz. F: The scientific community has not yet found an alternative to H that accounts for $\mathcal{D}$ and fulfills $\mathcal{C}$, and ¬F: The scientific community has found an alternative to H that accounts for $\mathcal{D}$ and fulfills $\mathcal{C}$.
3. Y has N values, viz. Y_i: There are exactly i hypotheses which explain $\mathcal{D}$ and fulfill $\mathcal{C}$. (H is one of them.)

A Bayesian Network Representation

We assume that T is conditionally independent of F given Y:

Conditional Independence

$$T \perp\!\!\!\perp F | Y$$



Once we know the value of Y, we won't learn anything new about the truth value of T if we learn the truth value of F.

The Prior Probabilities

To complete the Bayesian Network, we have to fix the values of $P(\mathrm{Y_i})$, $P(\mathrm{T}|\mathrm{Y_i})$ and $P(\mathrm{F}|\mathrm{Y_i})$ for all i.

1. The Priors y_i

$P(\mathrm{Y_i}) =: y_i$, with $0 \leq y_i < 1$.

This assignment reflects the fact that we do not know the number of alternative theories a priori.

2. The Conditional Probabilities t_i

The Conditional Probabilities

$P(\mathrm{T}|\mathrm{Y_i}) =: t_i$ are monotonically decreasing in i (and $t_1 = 1$).

This is plausible: The more alternative theories there are, the less sure we can be that H is true.

A natural choice is to apply the Principle of Indifference and to assume that $t_i = 1/i$, but for our purposes the weaker assumption is sufficient.

3. The Conditional Probabilities f_i

The Conditional Probabilities

$P(\mathrm{F}|\mathrm{Y_i}) =: f_i$ are monotonically decreasing in i (and $f_1 = 1$).

This is plausible: The more alternative theories there are, the less likely it is that scientists have not yet found one.

Calculating the Difference Measure $d(\mathrm{T},\mathrm{F})$

Given our "common-cause" Bayesian Network, the following holds:

Lemma

$d(\mathrm{T},\mathrm{F}) = \frac{1}{2\,\mathrm{P}(\mathrm{F})} \cdot \sum_{\mathrm{i}\neq\mathrm{j}=1}^{\mathrm{N}} (\mathrm{f_i}-\mathrm{f_j})\,(\mathrm{t_i}-\mathrm{t_j})\,\mathrm{y_i}\,\mathrm{y_j}$

Hence,

Theorem

If f_i and t_i are monotonically decreasing in i and if there is at least one pair (i,j) with $j > i$ for which (i) $y_i\, y_j > 0$, (ii) $f_i > f_j$ and (iii) $t_i > t_j$, then $d(\mathrm{T},\mathrm{F}) > 0$.

Discussion

- Note that the assumptions of the theorem are rather weak. If an agent assigns degrees of belief that satisfy them, then she will be rational to make the NAA.
- However, if someone believes, for example, that the number of alternatives has a fixed value, then F does not confirm T and the NAA has no pull. One could, for example, argue that the number of alternatives is infinite (i.e. that $y_\infty = 1$).
- Note, though, that scientists are often convinced that the number of alternative theories is rather small (without knowing the precise value). They are impressed by the difficulty to construct them. And this explains their conviction (supported by our analysis) that F confirms T.
- But is this line of thought convincing?

Underdeterimination

- Enter philosophy of science.
- According to the Underdetermination Thesis (UDT), there is always an infinite number of alternative theories that is consistent with a given (finite or countably infinite) set of data $\mathcal{D}$.
- So should a rational agent set $y_\infty = 1$?
- The NAA-er has two options to respond: (i) Argue that the constraints $\mathcal{C}$ reduce the number of alternatives to a finite number. (ii) Argue that UDT only shows that $0 < y_\infty < 1$ and that there is at least one (finite) pair (i,j) for which the assumptions of the theorem hold.

Two Other Worries

1. Often different scientists disagree upon what (i) the data $\mathcal{D}$ are the theory should account for, and (ii) which theoretical constraints $\mathcal{C}$ the theory should satisfy.

   Clearly, an NAA is only convincing if everybody agrees on $\mathcal{C}$ and $\mathcal{D}$.
2. The difficulty of the problem (or the ability of the scientist) should be included in the Bayesian model. This can be done in a straightforward way (see the paper). The present results hold for a fixed value of the difficulty of the problem.

   Note, though, that one will need evidence for the claim that scientists would find an alternative if there were one.

IV. Applications

1. String Theory

Despite extensive research, no empirically distinguishable alternatives to String Theory have been found.

1. Alternative approaches to achieve a unification of General Theory of Relativity and Quantum Field Theory have not led to new promising ideas and were eventually given up.
2. The scope of the program of Canonical Quantum Gravity (Carlo Rovelli, Lee Smolin et al.) is more limited.
3. General considerations suggest that even changes of very fundamental physical principles end at the Standard Model.
4. Different versions of String Theory are equivalent.

But the challenge remains: which reasons can be given for the possibility of a finite number of alternative theories (including the true or empirically adequate one)? Besides, there is disagreement on the conditions $\mathcal{C}$.

2. Scientific Realism

- We have defined the propositional variable T as follows:

  T has two values, viz. T: The hypothesis H is empirically adequate, and ¬T: The hypothesis H is not empirically adequate.
- That is, we (only) want to infer to the truth of all that H says about observable things.
- A scientific realist wants more, i.e. truth *tout cours*. Can we model this as well? And will the corresponding hypothesis also be confirmed?

2. Scientific Realism (Cont'd)

- Replace T by T'. T' has two values, viz. T': The hypothesis H is true, and ¬T': The hypothesis H is not true.
- F and Y remain as before.
- The independence assumption (i.e. $T' \perp\!\!\!\perp F|Y$) holds and our argument goes through.
- We ask: Which of the two propositions T and T' is better confirmed? Note that T' → T (but not T → T'), i.e. $P(\mathrm{T}) > P(\mathrm{T}')$ and $P(\mathrm{T}|\mathrm{F}) > P(\mathrm{T}'|\mathrm{F})$.
- Hence it is not clear whether $d(\mathrm{T}, \mathrm{F}) > d(\mathrm{T}', \mathrm{F})$. It needs to be explored under which conditions which proposition is better confirmed.

3. Inference to the Best Explanation (IBE)

- Under which conditions is an IBE justified?
- Replace F by F'. F' has two values, viz. F: The scientific community has not yet found a better explanation than H, and ¬F: The scientific community has found a better explanation than H.
- T (or T') and Y remain as before.
- The independence assumption (i.e. $T \perp\!\!\!\perp F' | Y$ or $T' \perp\!\!\!\perp F' | Y$) holds and the argument goes through.
- Question: Can this kind of reasoning be used to respond to van Fraassen's "bad lot" argument?
- Again, the answer depends on how good our reasons are that the true or empirically adequate theory is amongst a finite number of considered alternative theories. (Note the possible difference between ordinary reasoning and scientific reasoning.)

4. The Existence of God

- Recall N. R. Hansson's argument against the existence of God: So far we have no good reason for the existence of God, which provides a reason against the existence of God.
- Three propositions:
  1. T'' has two values, viz. T": God exists, and ¬T": God does not exist.
  2. F'' also has two values, viz. F": No one has yet found a good reason for the existence of God, and ¬F": Someone has found a good reason for the existence of God.
  3. Y'' has N values, viz. Y_i": There are exactly i good reasons for the existence of God.
- The independence assumption (i.e. $T'' \perp\!\!\!\perp F'' | Y''$) holds and the parameters f_i'', t_i'' and y_i'' are defined as before.

4. The Existence of God (Cont'd)

The Conditional Probabilities

$P(\mathrm{T''}|\mathrm{Y_i''}) =: t_i''$ are monotonically increasing in i.

Remember our (suitably adapted) lemma:

Lemma

$d(\mathrm{T''}, \mathrm{F''}) = \frac{1}{2\,\mathrm{P(F'')}} \cdot \sum_{\mathrm{i \neq j = 1}}^{\mathrm{N}} (\mathrm{f_i''} - \mathrm{f_j''})\,(\mathrm{t_i''} - \mathrm{t_j''})\,\mathrm{y_i''}\,\mathrm{y_j''}$

Theorem

If f_i'' are monotonically decreasing in i and t_i'' are monotonically increasing in i and if there is at least one pair (i, j) with $j > i$ for which (i) $y_i''\,y_j'' > 0$, (ii) $f_i'' < f_j''$ and (iii) $t_i'' > t_j''$, then $d(\mathrm{T''}, \mathrm{F''}) < 0$.

V. Outlook

Outlook

1. We have provided a Bayesian account of NAAs and analyzed under which conditions this argument-type is powerful.
2. Given that various assumptions have to be fulfilled, the acceptance of a proposed NAA will often be controversial.
3. Most crucial is the assumption that a non-zero probability can be assigned to the proposition that the number of alternative theories (including the true or empirically adequate one) is not infinite. The argument for this will depend on the specific application in question.
4. Open questions: Detailed case studies from science (e.g. string theory), NAAs in philosophy (e.g. the design argument), and (I am sure) more.

Thanks for your attention!

The talk is based on joint work with Richard Dawid (Vienna) and Jan Sprenger (Tilburg). Our paper is forthcoming in *The British Journal for the Philosophy of Science.*