

How Did You Guess *That*, Watson?

To the Reader

As the following reports will witness, Arthur Conan Doyle's Dr. John Watson was much underrated, and Sherlock Holmes the reverse, when the issues came to judging probabilities, and the use of those judgements in separating relevant from irrelevant evidence, inferring causation, and predicting the effects of their actions. Those who wish to address for themselves the problems that follow, before knowing Watson's solution and his explanation of his reasoning, should consider the problems and the evidence given before reading past the lines marked with

Watson's accounts are chosen to illustrate principles and methods of inference and discovery based on rigorous and systematic application of common sense principles, by methods common in computer science but which conventional statisticians generally refuse to apply.

Unlike the chronology of Sir Arthur's splendid stories, these are arranged from the simplest, and therefore least interesting, to the more complex.

The Author

1. The Fingerprint Twins

“I have been reading, Holmes, of this new device of Mr. Francis Galton’s for determining identity. Fingerprints he calls them. Each person, he says, has a distinct pattern or lines and sworls on each finger, unique as snowflakes. He claims he can match persons and fingerprints almost certainly merely by comparing five points on the finger and image. If he is correct, this should be a remarkable aid to detection. Apparently, by applying a powder, an image of any fingers that have touched an object can be obtained.”

“Piffle, Watson, another harebrained scheme by that cousin of the infamous Darwin. Galton is a man altogether too eager on behalf of himself and his class—have you read his self-congratulatory, *Hereditary Genius*?

“I agree, Holmes, that his writings are full of bad reasoning from selected statistics to cause and effect.”

“In any case, I have experimented with Galton’s idea in a real case, and it is demonstrably useless, especially compared with the powers of observation and deduction. I see, for example, that you and Mrs. Watson have had a disagreement recently.”

“How do you infer that, Holmes?”

“Your shirt is not so well boiled as usual, Watson, indicating either resentment or neglect on Mrs. Watson’s part, which can only be the result of some conflict between you, or quiet resentment on her part. What have you done?”

“I have chastised the laundry maid who boils my shirts, Holmes.”

“Well, you see my point nonetheless.”

“I see it very well indeed, Holmes, and, by the way, I agree with you about the quality of Mr. Galton’s argument. Whether Mr. Darwin is famous or infamous depends, I suppose on where you ask. In scientific circles he is justly famous; among the clergy, not so much.”

“He is refuted by the rose, Watson, the existence of which can only be explained by creation for the pleasure of the human eye and nose.”

“Perhaps, Holmes, but you fail to explain the thorns. But what is this experiment you made with fingerprints?”

“Lestrade presented the case to me. There are fraternal twins, Igor and Viktor, both newly released professional safe crackers. They both use tools and nimble fingers rather than explosives in their trade, but they never work together. Now both Igor and Viktor claim to have gone straight, but the Savoy Hotel safe was robbed last week, and Lestrade is sure that one of them did it.”

“Why is Lestrade so certain?”

“Because a faded picture of their mother was left at the scene!”

“Lestrade, like you, read Galton, and then obtained the relevant chemicals to take fingerprints. None were found on the gnarled combination lock, which Lestrade says is too rough a surface for the purpose, but prints of four fingers were found on the safe door. Lestrade took prints from each of the twins, to no use.”

“Neither twin’s hand matched the prints?”

“The reverse, Watson. The right hands of both twins matched the prints by Galton’s criteria.”

”I should have thought that impossible according to Galton.”

“Galton does allow that identical twins can have the same prints, but Victor and Igor are not such. I suppose it is more likely that fraternal twins will have nearly indistinguishable prints. But you now see how useless the prints are, Watson. Victor and Igor both deny the robbery, and since Lestrade is unable to make the case for one as against the other, and it is certainly not both, he cannot prosecute. Indeed, he cannot even obtain a warrant to search their respective premises for the missing money.”

“Are there any physical differences in the twins, Holmes?”

“Lestrade says they could nearly pass for identical. One is an inch or two shorter than the other, and Victor is right handed while Igor is left handed.”

“Then I think you know the solution, Holmes—it is just the application of one of your principles, probabilified.”

“What principle is that, Watson?”

“Excluding alternative explanations, Holmes, until but one remains. With which hand will a right handed man turn a combination?”

“His right, of course.”

“In all cases?”

“Not I suppose in every case. A dog might have bitten his right hand, and then he would use his left. But the probability is very high.”

“How high, Holmes?”

“Say 95 times out of a hundred, if you insist on numbers, Watson.”

“And where will the right handed thief place his left hand while fiddling the combination?”

“Most anywhere. In his pocket, behind his neck, against the wall, against the safe door.”

“And with what probability against the safe door, Holmes.”

“I do not know exactly, Watson. Perhaps 20 times in a hundred.”

“And for a left handed man?”

“I begin to see your point Watson.”

“Exactly, Holmes, he may brace his other hand against the safe. Since a right handed print was found on the outer door of the safe, we may infer with reasonable probability that the safe cracker was using his left hand to turn the combination, and therefore, also with reasonable probability, that the safe cracker was left handed, and therefore, assuming Lestrade’s hypothesis that one of the twins was responsible, that Igor is the one.”

“Since you are so insistent about numbers, Watson, can you say with exactly what probability Igor is the culprit.”

“I can on the assumptions: that Igor or Viktor did the deed, and that the probability that a right handed man turns a combination with his left and braces himself with his right hand is but 5 in a hundred, and, given that he braces himself, the probability of a left handed man bracing himself with his right hand whilst turning a combination is 95 in a hundred.”

“And what is the probability then, Watson, given a right hand print, that the safe cracker is left handed, and therefore Igor?”

“95 in a hundred.”

“You say then, Watson, that the facts, if such they be, that only 20% of thieves would so brace themselves, and the initial chances of Igor being the thief are fifty/fifty make no difference?”

“We need only calculate the probability that the thief is left handed, Holmes, for in that case he must certainly be Igor, no matter the initial probability that Igor was the thief. But we *know* the hand was braced, so the frequency of bracing is irrelevant. We know further that the bracing was by a right hand, and in only 5 percent of bracing by right hands is the person in question right handed, so in the remaining 95% of cases he is left handed. Applying this reasoning we obtain my conclusion.”

“The numbers in this case, Watson, are gratuitous. The reasoning could be done without them; I do the like in most of my cases.”

“I agree, Holmes, that the reasoning principles of science with probabilities are but numbered versions of common sense reasoning, sometimes of clever common sense reasoning, but nothing exotic or esoteric or ineffable. But there may come a time when the numbers matter, and common sense, although it has sound principles, is difficult to apply. Then, numbers, diagrams and algorithms can be genuinely of aid.”

2. Arsenic and New Silver

Holmes and I were vacationing at the country estate of a dear friend and former associate during my tour in the Raj, Colonel Blimp. Holmes had been not quite himself, and we had both thought a restful stay in the country might revive him--- and his powers. Alas it was not to be.

We retired after a handsome dinner of roast beef and Yorkshire pudding—although the estate was not in Yorkshire—followed by genial conversation about the career of Henry Ward Beecher, spoilt only by the Colonel’s young grandson, who, hearing Beecher’s name could not resist reciting a limerick:

*The Reverend Henry Ward Beecher
Thought the Hen a most marvelous Creature
The Hen, pleased with that
Laid an egg in his hat
And thus did the Hen Reward Beecher*

The only other oddity of the evening was that we dined with aluminum knives and forks, which had lately come into fashion sixty years after Napoleon began their use. I, at least, slept well. But at breakfast the next morning, Holmes looked weary.

“Watson, did you hear a ruckus last night?”

”Not a sound, Holmes. I slept the sleep of the innocent.”

“Good for you. Aha, here is the Colonel. I will ask him.”

Blimp entered, looking even more fatigued than Holmes, and took a dining chair, some tea, and nothing else.

“Was there a disturbance last night Colonel?”

“You might so call it, Mr. Holmes, but the term catastrophe would be more to the point. A servant girl came down with a vicious attack, as if she had been poisoned.”

“How is she?” I asked.

“Deceased as of five o’clock this morning. If it is not too great an imposition, Dr. Watson, will you examine the body? It will take considerable time for the officials to arrive.”

“Of course. Had she or others been ill?”

“Not immediately, but this is the third death among my staff in the last month. All of the other help have given notice. I will soon have to cook and clean and tend the animals myself.”

“Unthinkable,” said Holmes. I could on such occasions not tell whether he was sympathizing with the anxieties of the rich, or mocking them. I went on into a bedroom, where the poor girl was laid out upon a bed. I examined her body with great care, but of course also with solicitude. Then I returned to the dining room, where the Colonel and Holmes were now in company with the local constable.

“It is most definitely arsenical poisoning” I reported.

“In that case, Colonel,” Holmes asked, barely turning his head, “what are the sources of arsenic in the house?”

“There are but two, Mr. Holmes, a small supply kept near the kitchen for polishing silver, and in the gardener’s shack where it is used to poison weeds and occasional varmints.”

“Not a wise practice, Colonel, if you are fond of dogs, but that is beside the point. Who has the key to the gardener’s shack?”

“Only the gardener, Mr. Holmes.”

“Not even yourself?”

“Not even I”

“And who is responsible for the kitchen supplies, Colonel?”

“The cook.”

“Summon her, if you please.”

The cook duly appeared, a round and pleasant looking woman, by appearance and accent German or Austrian or Swiss. Holmes looked on her with immediate suspicion.

“Tell me, do you supervise the silver and other implements?”

She nodded, apparently understanding but either unable to speak English or too intimidated at the moment to do so.

“Show me, then, where you keep the cleaning materials.”

The woman took us downstairs to the scullery, where pots and pans lined up neatly, and a locked cabinet contained various cleaning materials. The woman produced the key and opened the cabinet.

“Do you alone possess a key to this cabinet?”

She nodded again.

Holmes produced his glass and examined the cabinet carefully. Finally he turned to the rest of us and spoke.

“Constable, arrest this woman.”

The constable took firm hold of the woman, who trembled nearly to fainting.

“Splendid deduction, Holmes” said the Colonel, “but how do you know she is the culprit?”

“There is an empty bottle of arsenic in the cabinet. I deduce that it has only recently been emptied, otherwise the container would have been discarded. It seems most likely that it was emptied into something the poor girl upstairs consumed.”

“One moment, Holmes,” I inserted, “may we see the silver?”

The woman pointed to chest of drawers across the room. The Colonel had at least a key to this furnishing, and opened the drawers for us. I looked at the implements while Holmes expostulated to the constable on his methods of detection.

“These pieces look new; so new as to be unused. And they are not silver”

“They are ” the Colonel said to me. “We sold the old silver only last week, and brought in new aluminum.”

“I assume, Colonel, that the old was carefully polished before it was sold.”

“Of course.”

I turned to Holmes and the constable. “Constable, Holmes, I very strongly suggest that you release the woman and arrest the gardener.”

They did not release her, but they did go to the gardener’s shack and force open the lock. Dimly lit and full of somewhat dirty tools, the shack held a workbench, and the workbench a tin of arsenic, half full.

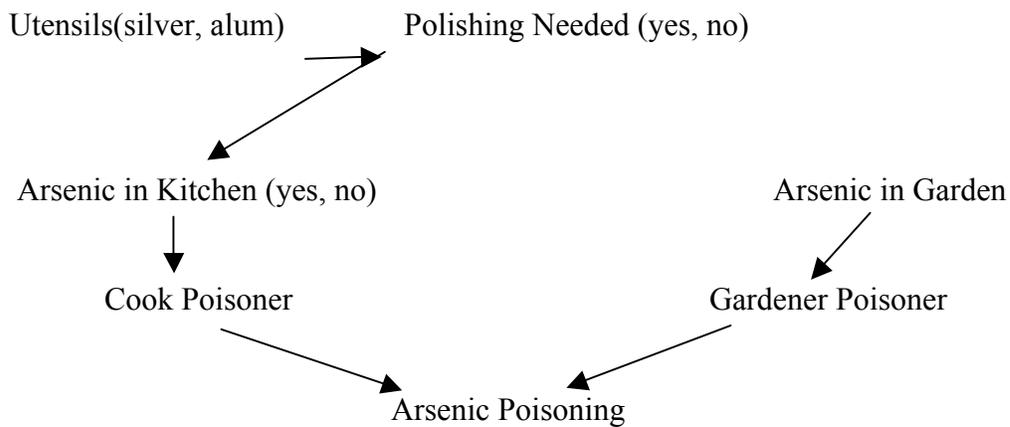
“Look around a bit for notes or a journal, any indication of records or plans,” I urged. Everyone complied, and under a loose floorboard Holmes hit upon a journal in which the gardener recorded his animus towards the entire household, based upon a history of overdrawn slights, not excluding having to hear the Colonel’s grandson’s mocking songs.

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Riding in a carriage back to the village to catch a train to London, Holmes recovered.

“Of course, Watson, if I had known of the silver and aluminum I would have immediately deduced the facts. But tell me how you did so?”

“I imagined a diagram, Holmes, and I can draw it in my notebook for you”



Then I considered probabilities that accord with the diagram: the probability of arsenical poisoning is determined by the determination of the cook to poison and the determination of the gardener to poison. The probability that the cook was determined to poison is determined by the availability of arsenic in the kitchen, the probability of which is determined by the need to polish, and so on. The fact we learned that silver utensils have been replaced by aluminum lowered the probability of arsenic in the kitchen. That, fact, together with the fact that the poisoning did occur, raised the probability that the source of arsenic was the gardener's shack. The rest is obvious."

"An admirable, if somewhat tedious explanation of your reasoning, Watson. Again, nothing but common sense."

"My very point, Holmes."

3. Bombs Underground

Visiting his rooms unusually late one evening, I found Holmes perturbed.

“Look at this note Watson. What do you make of it?”

He handed me a sheet of linen paper on which was written in a firm hand in dark ink:

To Mr. Sherlock Holmes,

For your amusement, and for mine, I have placed a small bomb in one of three London underground stations, Baker Street, Picadilly Circus, or Victoria, arranged to explode at midnight if not defused. The device is not profoundly hidden, and is simple to defuse—although it requires the cooperation of two persons. I afford you the opportunity to locate the explosive and render it harmless. Proceed, if you please, to a station of your choice. Once you have arrived there, I will send a message notifying you of one of the two remaining stations in which I have not placed a bomb. That is all of the information you will receive. If you notify the police of this circumstance, I will of course arrange to have the bomb detonate immediately.

Yours faithfully,

Moriarty

“What I make of it, Holmes, is that we should take ourselves promptly to the Baker Street Station. We have less than an hour.”

“But consider, Watson, the paper is definitely French, and from the absence of a watermark, the sheet has been made particularly for Moriarty. The ink, I see under my glass, is composed of a quite peculiar form of carbon. I believe we have the prospect of locating Moriarty through his French stationer”

“The bomb will have long since exploded, Holmes.”

“Indeed, Watson. Merely a thought.”

With that, Holmes threw on his cloak, forgetting even his cap, and we proceeded apace to the Baker street station by foot. Arriving, we were met by a lad who handed Holmes a note.”

“What ho, Watson. Here is a useless piece of information:”

To Mr. Sherlock Holmes,

The bomb is not at Victoria Station.

Moriarty

“Why do you say useless, Holmes?”

“The paper is cheap scrap, Watson.”

“But the question, Holmes, is where we should look for the bomb now that we know it is not at Victoria Station. Should we stay here and search the Baker Street Station, or should we go to Picadilly Circus Station and search there? We have time to do one or the other, but not both.”

“Oh, that. You could off to Picadilly while I search here, but the first note says two of us are required to defuse the thing, so we had best stick together. There are but two places, Watson, so the chances are even that the bomb is here as against in Picadilly Circus Station. Since we are here, here is where we should look.”

“I think to the contrary, Holmes. You have often assured me that, while evil, Moriarty does not lie, hence my inference.”

“Which is?”

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“We should both go to Picadilly, posthaste. I assure you, Holmes, our chances are better at Picadilly. Unusual as it is for me, I must insist.”

“How unlike you, Watson. But since it is actually a matter of indifference where we search, I will indulge you on this occasion.”

Holmes and I made the train and, arriving at the Picadilly platform, we soon located the device in a dustbin and disconnected its charge.

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I met Holmes for a short grog the next evening; he took tea instead, but his mood was congratulatory.

“We were lucky, Watson.”

“Indeed we were Holmes, but not merely lucky. We were twice as likely to find the bomb at Picadilly as at Baker Street. The probabilities, in other words were $\frac{2}{3}$ as against $\frac{1}{3}$.”

“Nonsense, Watson, the chances were, in the argot of the boys, even-steven. The matter is obvious, but to be sure of it I sent a description of the events over to Lestrade this morning. He has hired a new computer, a “statistician” the fellow calls himself, who specializes in calculating the probabilities of events. He came by himself to inform me that the chances the bomb was in Picadilly were fifty-fifty.”

“Whatever he may call himself, or others may call him, Holmes, his answer is wrong. Here let us play it out a few times.”

Drawing a piece of paper from Holme’s writing desk, I marked it “Picadilly,” “Baker,” and “Victoria.”

“Now, Holmes, you may play Moriarty and I will presume to play you, following the strategy I recommend. Choose a stop name at random and write a check under the name, keeping it hidden from me. I will choose a stop and tell you which I have chosen. You will then tell me, at random, one of the three stops that is neither the stop you have written nor the one I have chosen. Then we will see whether the remaining stop—neither the one I have chosen, nor the one you have told me is *not* the one you have written down—is the stop you *have* written down. I say that about $\frac{2}{3}$ of the time that will be so.”

“A bit awkwardly put, Watson.”

“But perfectly analogous to my strategy with the problem Moriarty presented us last evening. The point is that I must always give up my first choice in favor of the remaining possibility, the stop I did not choose and you did not reveal as innocent of the bomb. Let us try the game a few times.”

“Quite so.”

<u>Holmes writes</u>	<u>Watson chooses</u>	<u>Holmes Opens</u>	<u>Watson Changes to</u>	
Picadilly	Baker	Victoria	Picadilly	Watson wins
Baker	Baker	Picadilly	Victoria	Watson loses
Baker	Picadilly	Victoria	Baker	Watson wins
Picadilly	Victoria	Baker	Picadilly	Watson wins
Victoria	Picadilly	Baker	Victoria	Watson wins
Victoria	Victoria	Baker	Picadilly	Watson loses

“Egad, Watson, it would appear that you are correct. You do win $2/3$ of the time with your strategy. How often would I win if instead of switching, I stayed with my original choice?”

“Holmes, since the chances must sum to 1, you would win but $1/3$ of the time if you did not switch.”

“Very well, Watson, I am convinced, but I do not understand. What is the explanation?”

“Oh, there are several ways to poke at it Holmes. Let us begin with the most obvious. Under what conditions will I *lose* with my switching strategy?”

“It appears, Watson—no it is a firm deduction—that you will lose by switching exactly when your first choice is correct!”

“Indeed. And how often is that, Holmes?”

“What do you mean?”

“What fraction of times will I have chosen correctly initially?”

“Oh, a third of course. I have three choices and you have three choices, there are therefore nine possible pairs of choices, one of yours and one of mine in each pair, and in three of those nine pairs, the choices agree. It is like throwing a pair of three sided dice and asking the chance they come up the same.”

“Perfect, Holmes. So if I will have chosen correctly initially one time in three, and my switching strategy fails only if I have chosen correctly initially, then my switching strategy will fail one time in three, so will succeed two times out of three.”

“On average, Watson.”

“Yes, on average. But it is best to go with the averages.”

“You said, Watson, that there are several ways to explain the matter. Give me another.”

“Very well. Suppose I chose the Victoria. Then you agree that the probability that my choice is correct is $1/3$. I will use a symbol, and write it with the cooled ashes from your pipe:

$$Prob(My\ Choice = Your\ Choice) = 1/3.$$

Now, Holmes, what is the probability that the station revealed by Moriarty is neither My Choice nor Your Choice?”

“That is a certainty, Watson, one of the rules of the game.”

“So, to the pipe ash again, Holmes:”

$$Prob(Revealed\ Station \neq My\ Choice \ \& \ Revealed\ Station \neq Your\ Choice) = 1.$$

“And, do you agree that the probability of a proposition that is certain conjoined with a proposition that is uncertain is equal to the probability of the latter?”

“What do you mean, Watson.”

“For example, Holmes, what is the probability that $2 + 2 = 4$? ”

“That is a certainty, Watson.”

“And what, Holmes, is the probability that a flip of the coin in my hand will come up heads.”

“Since I see it is neither twice tailed nor twice headed, the probability must be _”

“So, to explain the principle you wondered at just now, what is the probability that heads will come up on a flip of this coin, *and* $2 + 2 = 4$?”

“That probability must be _, I suppose.”

“Exactly my point Holmes. Now to continue with the matter of the stations. The probability that My Choice equals Your Choice, *and* that the revealed station stop

is not equal to either of them is also 1/3, since the latter part of the proposition is certain and the former part has probability 1/3.. I will write again:

$Prob(My\ Choice = Your\ Choice \ \& \ Revealed\ Station \neq My\ Choice \ \& \ Reveled\ Station \neq Your\ Choice) = 1/3.$

Now, you remember hearing of Reverend Thomas Bayes?”

“He passed on more than a century ago Watson, although I understand some of the younger fellows still talk of him up at Cambridge, in between their Latin lessons and other silliness.”

“Just so. The Cambridge Bayesians, as they are sometimes called, follow him in this principle: *if one has a hypothesis and a piece of evidence, and one knows a probability of the proposition that the hypothesis and the evidence are true, and likewise the probability of the simpler proposition that the evidence is true, then the probability of the hypothesis **in light of the evidence** is the ratio of those two quantities.*”

“Somewhat hard to follow, Watson.”

“Could scarcely be simpler in our case, Holmes. Once more to the pipe ash:

$Prob(My\ Choice = Your\ Choice \ \mathbf{given\ that} \ Revealed\ Station \neq My\ Choice \ \& \ Revealed\ Station \neq Your\ Choice) =$

$$\frac{Prob(My\ Choice=Your\ Choice \ \& \ Revealed\ Station\neq My\ Choice \ \& \ Reveled\ Station\neq Your\ Choice)}{Prob(Revealed\ Station \neq My\ Choice \ \& \ Revealed\ Station \neq Your\ Choice)}$$

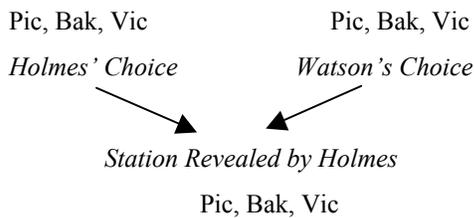
You have agreed, Holmes, that the number on top is 1/3 and the number on the bottom is 1. Now 1/3 divided by 1 is 1/3, which is, therefore the probability that my initial choice is correct, even after learning a station stop that is neither mine nor correct.”

“I suppose the reasoning is sound, Watson, but it is too complicated and seems to rest on an assumption made by the late Reverend Bayes that may be correct, but is not obvious.”

“Very well, a third explanation then, Holmes. This one is intuitive but incomplete. You will admit that whichever station stop you revealed to me in our games just now was influenced both by the station I chose initially and the station you chose initially, since the rules prohibit that you reveal either to me?”

“Yes, I suppose that is a kind of negative influence.”

“Good, then let us make a diagram of it with ash.”



“Capital, Watson! A causal picture as it were.”

“Yes, depicting two causally unconnected variables that have a common effect. I call a pair of arrows meeting at a variable a *collider*. Here then is the principle, Holmes:

If two variable features are such that the value of one of them provides no information about the other, and vice versa, and they influence a third variable, then given some value of the third, the value of either one of the two causes provides information about the value of the other cause.”

“So, you say Watson, that while your choice gave no information about my choice, nor mine about yours, once I revealed the innocent station, your choice provided you with information about my choice?”

“Exactly, Holmes: it told me that the probability that your choice was the remaining station was $2/3$.”

“This is very convenient, Watson, very easy to apply in manifold problems of detection. But I do not see how the diagram gives me the numbers, that is, that the probability of success on switching is $2/3$.”

“The diagram alone does not, Holmes. You must supplement it with Reverend Bayes calculation for the very numbers, although the diagram makes the calculation easier by specifying which feature depends on which other feature. That is for another time. But merely drawing the diagram that correctly represents

the causal situation can tell you what further facts are or are not relevant to which hypotheses in light of the evidence.”

“One further question, Watson. Suppose that Moriarity had told us the bomb was in one of four tube stops instead of one of three, and had then sent us the information about one of the tube stops that did not contain the bomb and was also not the stop we had initially chosen. Would the same strategy apply? Would we still be better off changing to one of the two other remaining tube stops?”

“We would indeed Holmes. The probability we would guess correctly by staying put would be $\frac{1}{3}$, and the probability that we would guess correctly by moving to one of the other two remaining possible tube stops would be $\frac{3}{8}$. So we would gain an eighth, or about 12.5% in chance, by switching.”

“Can you show me the calculation?”

“I could, Holmes, but Mrs. Watson is expecting me for dinner, so I leave it to you.”

4. The Case of Too Many Stations

“Well, Watson,” Holmes remarked as we strolled through Kensington Park, “it is a fine morning in England.”

“Here, at any rate, Holmes.”

“Ever cautious, ever pessimistic you are Watson. Let us presume until other evidence impinges that it is a fine day everywhere in the Kingdom.”

“Even in New Zealand?”

“Even there, Watson.”

Just then a shady looking lad stepped up from behind us and spoke to me.

“Would you be Sherlock Holmes?”

“I would not. He is the gentleman with me.”

“Gar, then this be for him.” Shoving a note into my hand without looking at Holmes, the fellow ran off. I gave the note to Holmes, who unfolded and gazed upon it with his intense, baleful eyes.

“A day ruined, Watson. Moriarity is at it again. See for yourself.”

I took the note, which read:

To Mr. Sherlock Holmes,

Since you had such excellent fortune with our previous engagement, I propose another. I have placed a small bomb, as before, in one of the first 100 London Underground stations. Please proceed to the stop of your choice, from which you may move from stop to stop as you please. Each half hour, at whichever stop you may temporarily reside, you will receive a message indicating a stop among the 100 at which the bomb does *not* reside. The messages will not repeat one another, no message will include the stop where you reside at the time you receive it, and all messages will be

the least informative consistent with these rules. You will receive 98 messages in all. The previous warning about contact with the police still applies.

Yours faithfully,

Moriarty

“Notice Watson, that the handwriting is not the same as before, but the paper is quite the same. Moriarty thinks he is a master of disguise even to his penmanship—he knows the difficult cases I have solved by analyzing scribbles-- but he errs in the vanity of his stationary.”

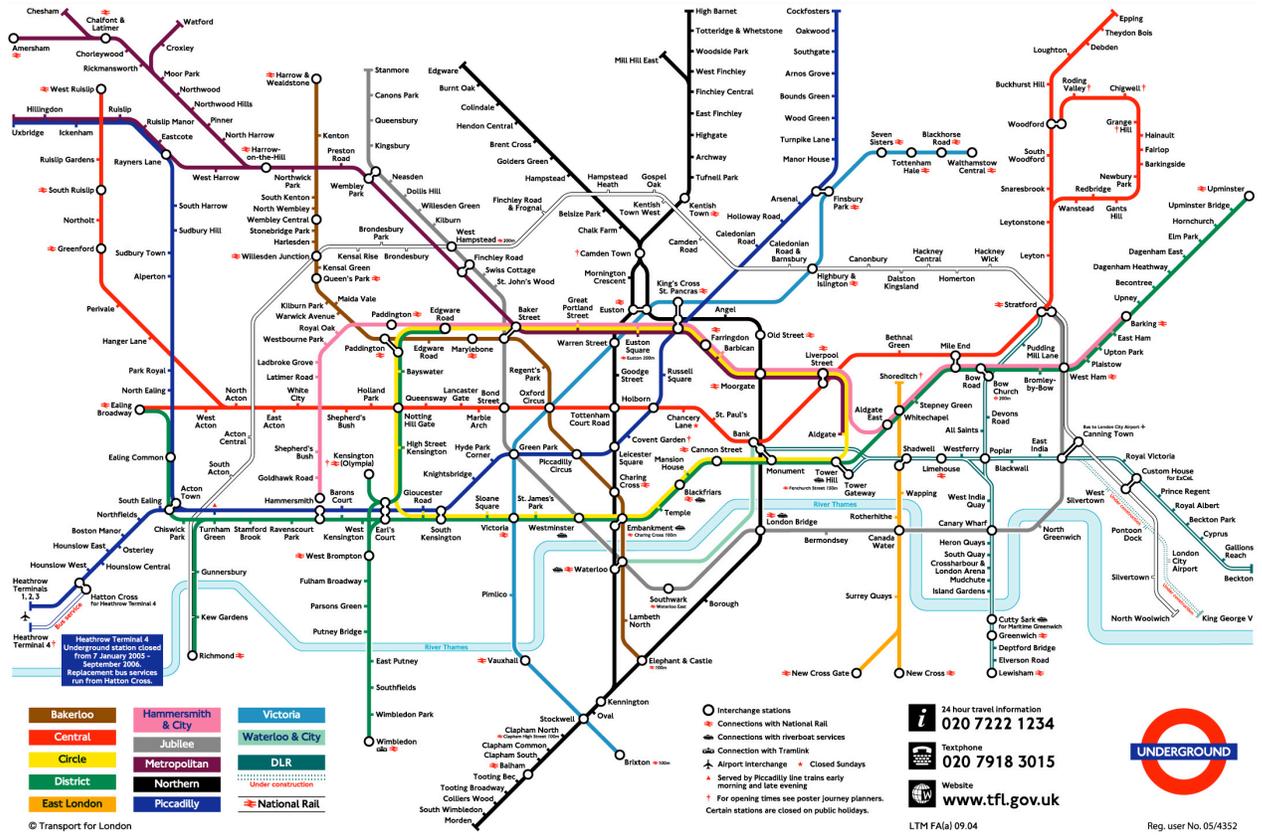
“Yes, but you know very well the value I place on French stationary as an aid to finding bombs. How *do* you propose to find the bomb, Holmes?”

“I have learned my lesson well from you, Watson, and of course I see the method. We will start at the closest tube stop, Kensington. Each time we receive a message we will take the train to the next stop that has not yet been eliminated by a message. With each move to a new station we will thereby improve our chances of finding the bomb, as compared with staying put. It is merely an iteration of your calculation in the case with 3 stops. In the end, we will have only two stops to choose between, and hence we will have, as before, a $2/3$ chance of finding the bomb. Of course, the endeavor will be quite exhausting, jumping here and there, on and off trains, for almost two full days and nights, without rest, and it will cost us more than a few pounds. But I suppose we are up to it.”

“I think, Holmes, that there is a much more restful strategy that has a greater probability of success.”

“What can you mean, Watson?”

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“I mean, Holmes, that we should proceed directly to a station is not too far from one of our domiciles—but not the Baker Street station—and take turns staying at that one station, as comfortably as possible, of course, receiving messages for the next two days, until the final message has been received. At that time there will be but two remaining possibilities, our station and one other, and we will proceed to the other station and find the bomb, almost certainly. With a grog, a book, your pipe, and relief, say, every four hours, it should be quite endurable.”

Holmes agreed, and we took the train to a stop close by Baker Street Station, and waited for the first message.

“Egad, I shall need a very fat book, Watson!”

“Mr. Darwin’s old but still controversial book on species is rather lengthy; then again, there is Dr. Freud’s new book on dreams.”

“Do you, as a medical man, put stock in Freud’s accounts of hysteria and neurasthenia and such, Watson?”

“His accounts in the medical journals change almost by the week. Would you diagnose yourself as neurasthenic, Holmes?”

“I suppose I have a neurasthenic tendency; I am sometimes quite completely exhausted as you know, and require considerable periods to recover.”

“Do you still take cocaine?”

“Only rarely; it gives me the illusion of powers even greater than those I truly possess, and no man should risk become a reflexive Deist.”

“Well, Dr. Freud recommends the stuff, Holmes, for almost any disorder or discomfort. And your neurasthenia he attributes to your private erotic activities.”

“More imagination than science, Watson, and none of his damnable business in any case. But enough of this chit-chat. Why will staying at one station do and why if we are to stand duty at but one station should it not be the one most convenient, namely Baker Street Station?”

“Have confidence in me, Holmes, and I will explain when we are less distracted.”

Two days later we had received messages eliminating each station except that which we had steadfastly manned, and one other, Baker Street itself. Proceeding to Baker Street Station, and looking about, we soon located the bomb underneath a bench, and disarmed it.

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“Well Watson, we were lucky again, although, as you have shown me, the chances were 2 in 3 that we would find the bomb.”

“I am sorry to differ slightly Holmes. The chances were 99 in a hundred.”

“You claim, Watson, that at the end you were virtually certain the bomb was at Baker Street Station? How can that be?”

“Let us go through the reasoning again, Holmes, as before. What was the chance that the station we chose in the first instance held the bomb?”

“One in a hundred, I would suppose.”

“Indeed, and the chance therefore that the bomb was *not* in the station we chose?”

“Ninety nine in one hundred, of course.”

“And in what circumstance would we be wrong if, at after the 98th message, we remained at our original stop and did not switch?”

“As before, Watson, exactly in case the bomb had been placed at our original stop.”

“An event that has but one chance in 100, as you have just allowed, Holmes.”

“Indeed.”

“So the chance was 99 in 100 that the bomb was in the other remaining station, Baker Street, where we located it. There was absolutely no need to go rushing about from station to station after each new message, and indeed the execution of such a plan might have lowered our chance of success, although not a great deal in this case.”

“But why, then, Watson, did we not take our stand initially at Baker Street Station, which would have been more convenient—although as the matter turned out, would also have resulted in an unwelcome explosion?”

“Because, Holmes, I assumed that if we could devise the most efficient strategy for the problem, then so could Moriarty, and, further, because he holds you in such high regard, Moriarty might well have concluded that we, too, could hit upon the most efficient strategy. Taking into account his knowledge of your sometimes languorous disposition, I thought Moriarty might therefore have concluded that we would wait out the messages at the stop most convenient to us, Baker Street, and then at the end we would switch to the single remaining station. If so, he would place the bomb at Baker Street. Therefore, it was imperative that we *not* initially select Baker Street.”

“Rather a he-thought-we thought-he thought, Watson.”

“Just so. Fortunately, Moriarty stopped one level before we did.”

5. The Three Penny Opera

Holmes and I were walking to his tobacconist when he stopped and pulled me across the street, so rapidly I was nearly run over by a drayman. There, on the corner was a kind of minstrel show. The Master of Ceremonies introduced himself as Herr Weil, and in accent meant to be German, but noticeably faked, announced that he would flip three coins—one of silver, one of iron, and one at least painted gold-- in the air to land in an open pan, following which he would turn his back on the pan, don a blindfold, and would, or would not, sing all three parts to the trio “Zitti Zitti” from the Barber of Seville. His Sideman would post a prediction as to whether the MC would sing, or not, and take bets against his prediction from bystanders. So, if the Sideman held up a sign saying “SING” which the customers could see but the MC could not, and a customer held up a finger, the customer would lose a shilling if the MC sang, and win as much if the MC did not sing. And if the Sideman held up a sign saying “NOT SING: and the customer held up a finger and the MC sang, the customer would win a shilling, and if the MC did not sing, the customer would lose a shilling. The MC assured the crowd of passersby that if he won, all proceeds would be given to a fund for the poor.”

“You see, Watson, the coins are the deception. Pure show. There must be a confederate in the audience who signals the MC whether to sing or not according to the Sideman’s prediction. Perhaps he does so by coughing, or tapping a foot. We have only to watch and he will soon be located. Then we can summon a constable and put these cheats out of business. Besides, I think the MC to be a communist.”

“Very good, Holmes. You have a keener eye for these matters. I will watch the coin flips and perhaps jot them down in my notebook, and keep an eye on the Sideman as well.”

Holmes watched three plays and suddenly shouted “Scoundrel!” A constable came running and, recognizing Holmes, followed his instruction to haul away a

poor plain fellow who had coughed twice. The show went on nonetheless and I continued to watch.

After another half dozen plays, with new customers arriving, Holmes shouted again for the constable, and had him haul away a fellow who had been tapping his foot—I thought to the music, but I said nothing.

After another dozen plays I was growing weary of the game, but Holmes was keen to find another confederate. At last, I stopped him.

“Holmes, there are no confederates. Do have the constable release the men he has taken.”

“And how would you know that. Watson?”

“We have watched the game now for more than an hour Holmes, with more than 20 plays. I have noticed that the coins appear to be fair: each comes up heads about half of the time, and that one or two come up in any particular way does not change the chance of heads for the third. I have noticed further that the MC sings in about one quarter of the plays. I have noticed, finally, that if the MC sings, and the iron coin is heads, the silver coin is very likely to be heads and the gold coin tails, where as if the MC sings and the iron coin is tails, the silver coin is very likely to be tails as well, and the gold coin heads. If the MC does not sing, such arrangements do not occur.”

“And so?”

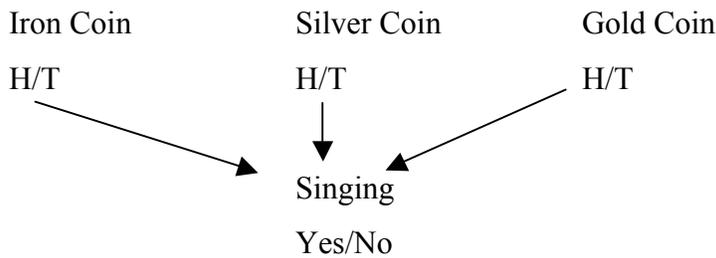
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“And so, Holmes, from this information I deduce that the Sideman and the MC are following a rule: *if the iron and silver coins agree, and the gold coin differs, then sing. Otherwise do not sing.* No confederate is needed.”

“And how does this follow, Watson?”

“Recall, Holmes, the causal graph we considered in the case of the London Underground. Supposing the outcomes of the flips determine the singing, show me on this page of my notebook how you would diagram the relationships.”

“Something like this, Watson:”



“Exactly, Holmes. And would you expect the side of the iron coin to give you any information, by itself, of the side of the silver or the gold coins.”

“I would not. Unless there is a cheat, they should be independent.”

“And so it is in the sample I have observed, Holmes. But now, remember our cases with the Underground, and the principle we applied there.

If two variable features are such that the value of one of them provides no information about the other, and vice versa, and they influence a third variable, then given some value of the third, the value of either one of the two causes provides information about the value of the other cause.”

What holds for two causes holds as well for three. If your diagram is correct, and you knew in a particular case that the MC sang, would information about the value, heads or tails, of the iron coin tell you anything about which way the silver and gold coins had landed?”

“Indeed, it would. I should know that the silver coin had the same side as the iron, and the gold coin the opposite one.”

“Just so, and exactly what I found in the sample I observed. So the best explanation is that whether the MC sings or not is determined by how the coins land, and specifically that he sings if iron and silver agree and gold disagrees.:

“So it’s the collider, not the confederate, Watson.”

“That is the best explanation, Holmes.”

“But it appears to be that your inference applies your principle in reverse, Watson. Rather than inferring the associations from the causal relations, as the principle states, you have inferred the causal relations from the associations.”

“Quite so, Holmes. The entire business could be due to chance, but the patterns of association we have seen, if robust, can only be explained as I propose.”

“Remarkable, indeed, Watson. But of course you had the advantage of knowing that the coin tosses occurred before the singing, so the direction of influence was not in question.”

“Yes, I had that advantage, Holmes, and in many cases it is very useful to know the sequence of events. But in this case it was supererogatory, quite unnecessary.”

“You are a mathematical peacock, Watson. I don’t believe for a moment that you could, on any reasonable assumptions, know the direction of influence without knowing the order in which events occur.”

“Another time, then, Holmes, should an appropriate case arise to illustrate the point. In the meanwhile, will you see that the poor blokes are freed?”

6. The 40% Solution

I found Holmes at work in a corner of his rooms reserved for chemical experimentation. He was siphoning some foul looking liquid from one flask to another.

“Good morning, Holmes. I see you are in a fit of chemistry.”

“Yes, Watson, preparing a new reagent to detect alkaloids.”

“What, alkaloids! Have there been poisonings of some kind?”

“None have come to my attention. I am distracting myself.”

“From what?”

“From a problem, Watson, set me by the French government and recorded in the fat file you see on my desk. When I have finished this concentration, I will tell you about it.”

When the siphoning was completed, Holmes carefully washed his hands and then, seated, took up a pipe and began to explain, with a bemused look.

“The problem is ridiculous. The French are convinced there is a ring of Afghan fiends operating who plan to destroy Msr. Eiffel’s new tower. The French police started with one suspicious name and have watched every meeting between him and others for several months, every meeting between those who met with him and still others, and so on.”

“Soon they will have watched everyone in the world, Holmes.”

“Not quite, but altogether they have observed two thousand people and more than a thousand meetings.”

“What transpired at those meetings, Holmes?”

“That is the amazing part of the problem, Watson. All of the conversations overheard were in an obscure Afghan dialect. The French know which persons met with which others, and when and where they met, all of which is recorded in

the file on my desk; the conversations overheard were copied down so well as could be expected, but the French have absolutely no idea what was said at any of the meetings.”

“Very strange. And the meeting places?”

“Completely ordinary. Cafes, restaurants, happenstance on the street. Nothing to distinguish them from meetings one might have with one’s friends or relatives.”

“Quite likely most of the meetings *were* between friends and relatives and completely innocent, Holmes. It would be a poor conspiracy indeed that required two thousand participants and a thousand meetings in Paris.”

“Indeed, Watson. I suspect there are only a handful or two of actual conspirators. in the lot, and the French assure me the gang has fewer than 100 members, and probably less than that. The preposterous thing is that the French Securite’ hopes that I will be able to distill them from the rest merely from these meeting records. I am a detective, not a thresher to seek a needle in a haystack.”

“But you *are* a kind of chemist, Holmes, and the problem is essentially chemical, not physical.”

“What an obscurity, Watson! What can you mean, man? Speak plainly.”

“In chemistry the problem is often to increase the concentration of a desired chemical in proportion to a less desired one. So in distilling, for example in making alcohol, one starts with the mixture of water and alcohol produced by the action of yeast on sugar, a mixture in which the proportion of alcohol to water may be very low, and seeks, by distilling, to produce a mixture in which the proportion of alcohol to water is much higher.”

“I do not see the analogy, Watson.”

“But it is plain, Holmes. You have a mixture of people, the innocent and the evil; the number of people is quite large, and the number of evil comparatively low, and the ratio of evil to innocent is something on the order of 100 in 2000 or 5%. The goal is to find a way to select from this large body of people a collection in which the concentration of evil is much higher. We seek to find a property, a mark, that identifies a collection in which the evil are more frequent.”

“To what purpose, Watson?”

The French police cannot trail and investigate two thousand people, and if they did and their effort were discovered and made a subject of the press, the Republic might fall. But they might well investigate fifty, and if half or more of those were conspirators, their investigation would be well served.”

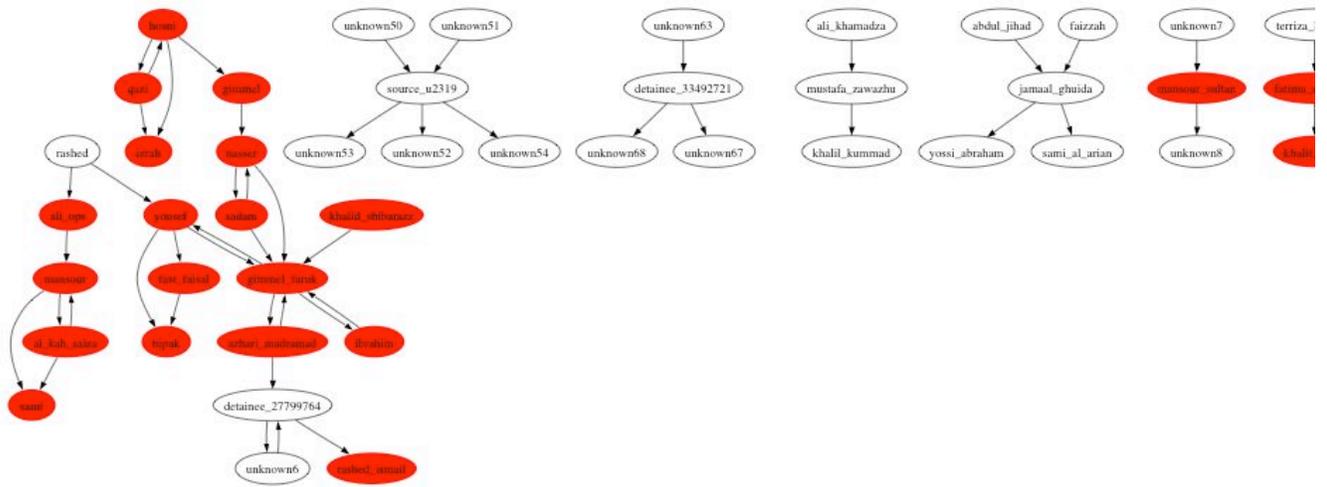
“But, Watson, they would have missed many conspirators.”

“Indeed, Holmes, they would. Our problem is exactly as with distilling—you leave some of the alcohol behind in the vat in order to obtain a liquid with a higher concentration of what you desire. Except that if one were to detain twenty five of seventy five conspirators, the twenty five might be persuaded to lead to the rest, or to many of them.”

“Quite a novel perspective, Watson, but I think for all your talk of discovery as chemistry, you lack a retort and a distilling column. I have given the French data to one or two professors who specialize in what they call “social network analysis.” Their results are not useful.

“Why not, Holmes?”

“Some of them merely make a diagram without arrowheads connecting every pair of individuals who have met with one another. The result looks for all the world like the product of a badly managed Italian kitchen—a ball of spaghetti. Some of the ‘analysts’ then “thinned” the spaghetti ball by allowing connections only among those who had met numerous times. The result was simply the social butterflies. Others thinned by what they called “centrality measures.” I did not understand these measures, but I think they amounted to assigning to each individual the number of individuals who were connected only through him. By and large the central characters were the social butterflies once more. I can scarcely tell the French to focus on those who are most sociable. Here is how their diagram looked after they twice eliminated the pairs who communicated least frequently.”



“Good work, Watson. A 40% solution, up from less than 5% and capturing at least a quarter of the conspirators. How did you do it?”

“Common sense, Holmes, supplemented with two weeks of calculation. If only the Difference Engine of that mountebank, Babbage, really worked, I think the thing could have been done in a few minutes.”

“Yes, but how, Watson, how did you do it?”

“Teams of conspirators, especially those involved in quasi military projects generally have a chain of command, possibly quite short: commanders, lieutenants and privates. Similarly, there is a reverse chain of reporting of difficulties or results in preparing for or executing a plan. So, in our problem, it would be helpful to identify chains of command or chains of reporting. That is all that I did.”

“Yes, but will you not also identify other innocent chains that way: the wholesaler meeting with the retailer meeting with the client, and so on?”

“Of course one will, Holmes, just as you suggest. But we should expect the innocent to be less often involved in relationships that exhibit such chains than are the conspirators, and that is indeed what the French found.”

“But how did you identify such chains?”

“With another hypothesis, Holmes, about conditional probability in such chains: if X commands Y, and Y commands Z, then the chance that Y meets with Z increases if X has recently met with Y.”

“Fair enough, but how can one measure the chances? They are, after all, not like colors or sexes or clothing. No one, not even the infinitely observant Mycroft, has ever seen a chance.”

“What you say is fair, Holmes. I estimate the chances this way: We have reports in the French file from almost half a year, 180 days to be exact. I take meetings on occurring on the same day as simultaneous—a day is my “window of time” as it were. For each pair of persons in the files, call them X and Y, I counted how many days they met and divided that number by 180, giving me an estimate of the chance of X meeting with Y. Then, for each third person, Z, I counted how often X and Y had met given that X had met with Z on the previous day, and divided again by 180, giving me an estimate of the chance of X meeting with Y given that X had met with Z on the previous day. I did the same with Y in place of X. If the conditional probability of X and Y meeting given that X has met with Z on the previous day is sufficiently greater than the unconditional probability, I infer that there is a chain of command, or a chain of reporting information, $Z \rightarrow X \rightarrow Y$ ”

“Clever enough, Watson, but what if a lieutenant had the habit of meeting with his subordinates on the very same day he received orders from his commander?”

“Yes, that is a possibility, and to take account of it, I did the entire calculation again, taking windows from noon to noon instead of from midnight to midnight. I then estimated the existence of a chain from the average of the differences of the conditional and unconditional probabilities by the two ways of segmenting the times.”

“Very well, and this produced the results I passed to the French?”

“I am afraid not, Holmes. It produced about 250 names. I was forced to supplement the procedure another way, by using the reports of what was said at the meetings.”

“What, Watson! You did not tell me that you understand Afghan languages!”

“Not a word, Holmes, but I can count. I found words that occur in only a subsample of the meetings. Any words that occurred in all or almost all of the meetings I discarded. I was left with but a few hundred words. I then redid the entire computation I have just described, but with one further twist. In counting cases in which X met with Y more frequently shortly after X met with Z, I only counted cases in which the meetings of X and Y shared a least one of these exceptional words with the meetings of X and Z. That was how I determined the list I gave to you. By the way, the 25 whom the French identified as conspirators were in both the larger and the smaller lists. So by paying attention to the words, many innocents were eliminated from my lists.”

“Quite a lot of computing, Watson. If there had been ten thousand suspects you would be occupied the rest of your life, unless Babbage’s fantasy were to come true. But tell me one thing, Watson. In order to come to conclusions, you must necessarily have set a time limit for what counts as “recent” previous meetings, and you must necessarily have chosen some value for the difference in unconditional and conditional probabilities to be sufficiently large to warrant suspicion. How did you know which values of these parameters, if I may so call them, to choose?”

“Oh, I have a confession, Holmes. I have done such a problem before, on a much smaller scale, for the Danish, who were worried about a plot to destroy their cheese. In that case I tried several parameter values. In the present case, I used the values that worked best for Denmark.”

7. The Case of the Five Candles

Quite unusually, Holmes appeared early one evening in my surgery, as I was completing examination of my last patient of the day. He looked unusually eager, almost overwrought, and fierce eyes urged me to be done with my client, while his thin lips let not a word through.

“Now, then, it is a pleasure to have you here, Holmes, but what brings you?”

“Unbeknown to you, Watson, Lestrade and I have been watching a gang of criminals for many months, he with the aid of his earnest but unreliable police, I with the aid of my devil-may-care yet entirely reliable Baker Street Irregulars.”

“I do not understand, Holmes.”

“The enemies of Britain have been preparing a plot, I believe, to destroy the Tower of London. So that they cannot be apprehended together, they have been passing orders by placing candles in their windows. I think tonight the order will be given to the operatives to attack.”

“So why not have Lestrade arrest them?”

“Lestrade has been waiting in hopes of developing more information, but he says that tonight, even with our aid, he has only enough men to secure one house. So, we must decide which house can be seized to choke off communications.”

“Well, Holmes, why not seize whichever house begins the communications?”

“Because, my dear Watson, we do not know which house that is, or if the orders originate from a single house.”

“My, my, quite a conundrum. How many houses are there?”

“Five.”

“And they are all on Tooting Road?”

“Indeed, Watson, and oddly all within sight of one another.”

“And what do you know, Holmes, about the communications among the five?”

“I have put the Baker Street Irregulars upon it, and they have collected a great many records for many nights, one boy attending to each house, recording whether a candle burned in the window, or not.”

“Then it is easy, Holmes. The sequence of times at which the candles light up will tell us who is signaling whom, and the order of communications.”

“Not so, Watson. The Baker Street boys, you know, do not possess watches. They can only mark times by the hourly notes of the local clock.”

“You are asking, Holmes, how to discover a causal pattern from observations without any experimentation, and without any knowledge of the order of occurrence of events.”

“Just so, Watson. I have the records the Baker Street Irregulars have sent to me. I can make nothing of them, and Lestrade, who has hired a new computer—a “statistician” he calls himself—says he cannot either, and, besides, the computer says that it cannot be done except possibly by a long sequence of calculations he calls “regressions,” a method due to Mr. Galton and his associate Professor Pearson, whom I believe you know. But you say you have new methods of causal reasoning. Perhaps you can make something of the records.”

“Dear Holmes, for reasoning about causes I have no new tool beyond common sense. I only try to make it consistent and rigorous. And, yes, I know Pearson, an excellent calculator, an inadequate philosopher, and a friend of the poor.”

“I will bear your summary of his character in mind, should I ever meet him. In the meanwhile, Watson, take a look at the Boys’ numbers, if you will.”

“Egads, Holmes, there are a hundred of them!”

“Yes, the Boys have been eager for the task and the conspirators have been at it since the New Year. Each column is labeled by the number of the house.”

H1	H2	H3	H4	H5
"0"	"0"	"1"	"1"	"1"
"0"	"0"	"0"	"0"	"0"
"0"	"0"	"0"	"0"	"0"
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“Well, Holmes, that certainly is a job. Let me take the papers home for a few hours. I must think.” And with that, Watson took his coat and hat and stick, and left in a flurry.
“Hurry, Watson: The fate of the Kingdom may depend upon you!”

.....

Later that evening, I took a horse cab to Holmes quarters. I found him at his study table, examining bits of wax through a glass.

“Watson,” he said, “I have hit upon the answer. I recalled that the lads had returned bits of wax that had dripped over the window sills of the five houses. They wisely labeled each bit with the house from which it came. I see that house number 4 has sometimes used a fine candle which I deduce is of Italian origin, while the other samples are all of ordinary tallow. It follows that the occupant of house number 4 is a foreigner, and undoubtedly therefore the ring-leader. If we capture him tonight, the matter will be resolved. So to house 4!”

“Not so quickly Holmes. Do I understand you correctly that Lestrade has only sufficient men to occupy one house?”

“Yes, and time is of the essence.”

“Then send to him, by the fastest messenger you can, this message: that he should take command of house number 3, Tottigham road. That is the best wager to block the signal from getting through. We can go directly ourselves. I have my revolver in my coat.”

“And you know this, how, Watson?”

“From the numbers you gave to me, Holmes.”

“Pray, explain.”

“There is no association between nights when houses 1 and 2 burn a candle—knowing that one of them is, or is not, burning a candle does not change the probability that the other does. They are, in the phrases favored by Lestrade’s statistician, independent. But the lights of every other pair of candles are associated—if any one is lit, it is more probable the others are, and if anyone is dark, it is more probable the others are also. The association is not perfect, but it is very strong. And there are two more complex facts. If we examine only the cases in which house 3 is lit by a candle, we find that the illumination of houses 1 and 2 is then associated--

“Are you sure, Watson, that you have the right house number? The wax says other”

“There is no certainty in this life, Holmes, save that it will end and that the King will levy taxes in the meanwhile. You have my best guess, and it is made with good reason.”

Arriving at 3 Tottigham Road just as Lestrade with two officers appeared, Holmes and Watson entered the premises in their company and secured a foreign rogue who was only at the moment preparing to light a candle. Upon his arrest, he gave the names of his confederates, and the members of the ring of the five signs were soon in gaol.....

Celebrating in Holmes' rooms the next week with a well earned reward of flutes of champagne, Holmes remarked, with languor:

“Now Watson, let me tell you what Lestrade uncovered about the plan of communication of the five candle signs by forcing confessions from the denizens of the houses.”

“Wait, Holmes, let me tell *you*.”

“How would you possibly know Watson—you may have guessed well enough to stop the signals last week, but how would you possibly know their entire plan of communication?”

“From the data you provided me Holmes, although I don't suppose I really *knew*; but I did more than guess.”

“Well, then, out with it Watson. I wait with bated mind.”

”Indeed. The plan was very simple. Either the rogue in House 1 could initiate a signal, or the rogue in House 2 could, quite independently. In either case, if the fellow in House 3 saw a lit candle in the window, he in turn lit a candle himself. The fellow in House 4 lit a candle if he saw one lit in the window of House 3, and the fellow in House 5 did the same. Either of their candles, in House 4 or House 5, was a call to some villainous action, and together they provided a certain redundancy.”

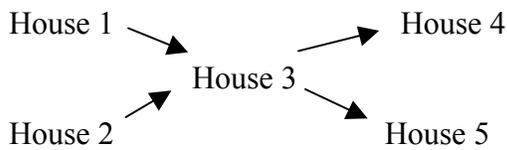
“Capital, Watson! And how indeed did you divine the truth from the boring table of naughts and ones I gave to you?”

“Simple enough, Holmes. The true structure of communication, which I have just described to you implies several things: *First*, there will be no association of occurrences of a light in House 1 and a light in House 2; the frequency with which both occur together will be just about the same as the multiple of their separate frequencies. That is what is meant by “independence” in the probabilists' sense. *Second*, the plan of the five signs I have described implies that the appearance of candles in the other houses will not be

independent of each other, or of candles appearing in House 1 or House 2. Third, the plan implies that if we examine only the cases in which the candle in House 3 is lit, the frequencies of lit candles in Houses 4 and 5 will be independent, and the same will be true if we consider only the cases in which the candle in House 4 is dark. That is what it means when the probabilists say the lighting or absence of lighting of the candles in Houses 4 and 5 are *independent conditional* on the state of the light in House 3. Finally, in the same way, the plan of the five signs I propose implies that the light in House 4, and likewise House 5, is independent both of the light in House 1 and the light in House 2, conditional on the state of the light in House 3.”

“Watson, this is both confusing and not to the point. How did you *discover* the plan.”

“Give me a further chance with a diagram, Holmes” said Watson, wetting a finger and dipping it in some cooled pipe ash. “Look here, this is the plan of signs I infer.”



“Now notice Holmes, that there is no connection, no signaling, between House 1 and House 2. If one of these houses lights a candle or not makes no difference to whether the other does.”

“Seems plain enough.”

“Notice further, Holmes, that House 1 and House 2 can communicate with House 4 and House 5, but only via a signal to House 3.”

“Quite obvious, Watson.”

“So Holmes, if you were to intercede to prevent House 3 from lighting a candle, whether a candle was lit in House 1 or House 2 or not would have no influence on House 4 or House 5. And likewise, if you kept a candle always burning in House 3, House 1 and House 2 would be unable to send a message according to this plan to Houses 4 and 5. And in these cases the events in Houses 1 and 2 would be independent in probability of the events in Houses 4 and 5. ”

“Just so.”

“But, Holmes, surely it makes no difference whether you *intercede* to prevent House 3 from showing a candle, or instead merely look at all of those cases in which the candle in House 3 is not illuminated. Surely, among these cases in which the candle in 3 is constant, variations in lighting in House 1 and House 2 must be independent in probability of the variations, if any, in the lighting of Houses 4 and 5.”

“Strange, but I suppose that is right”

“And the same reasoning shows that if we consider cases in which the state of illumination of the window in House 3 is constant—either absent in all cases or present in all cases—then the events in House 4 will be independent of those in House 5.”

“Watson, confound it, this is all too tedious. I grant you your independent this and conditionally independent that. So what! How did you uncover the plan?”

“You see, Holmes, the pattern of independence relations and conditional independence relations I have described is unique to the plan I have represented in the diagram. No other plan of communication among the five houses has these independence relations and only these.”

“None? In all of the fantasies of mathematicians, in all of the Forms in Plato’s Heaven, not one other plan, Watson?”

“Just so, Holmes, no other.”

“So, finish, how did you discover the plan?”

“In two ways, Holmes, which I will briefly describe. Forgive me for skipping some details; I know numbers bore you.”

“Nearly to death, but your capacity for discovery fascinates me.”

“In one way, I used ideas of the late Reverend Bayes, F.R.S., of whom you know. I started with the hypothesis that there is no communication among any of the Houses, and...”

“Absurd! Why waste your time?”

“Time is not wasted on a sound procedure, Holmes. To simplify, I computed the probability of the data on that hypothesis, and on every hypothesis that added to it one signal in any direction between and one pair of houses. I then computed a new probability for each of these hypotheses by Reverend Bayes method, and chose the most probable among them. I continued in this way, allowing myself also to remove connections I had

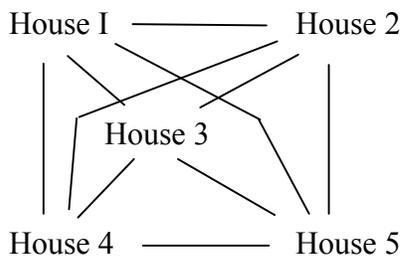
previously postulated, until I could find no further improvements in probability. The result is the diagram I have drawn in ash for you.”

“That seems very complicated, Watson.”

“A bit, but the actual process was more complex still. I have lied a little, kind lies, I assure you.”

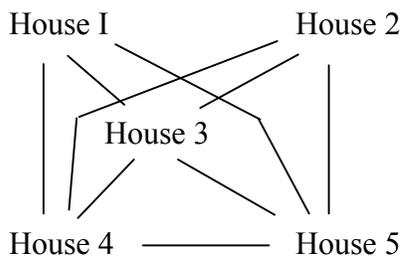
“So that is all, that is all that you did before you came rushing back to my rooms?”

“No, Holmes, I checked by another method, which is simple to describe. First I drew a diagram in which all of the houses were connected by lines without direction, like this:



“Absurd again, Watson, you have everyone signaling everyone, or something.”

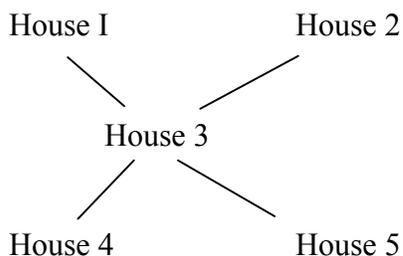
“Holmes you have no more patience than a French probabilist too opinionated to wait to turn a page before he condemns a procedure half described. Watch now. For each pair of Houses connected by a line in my diagram, I tested whether they are independent in probability, that is, whether the state of the window of either one, illuminated or not, provides information about the state of the window of the other. If I found no association, as with House 1 and House 2, I removed the edge between them in my diagram. The result of this step was so:”



“Next, Holmes, I asked of the data for each pair that remained connected by a line, for example, House 1 and House 5, and each house likewise connected by a line with one of them, for example House 4, whether the pair are independent conditional on the state of the third, that is, whether House 1 and House 5, for example, are independent conditional on the state of House 4, whichever the latter may be, illuminated or dark. In this case there is no such conditional independence, but in the case of House 1 and House 5 conditional on House 3, and likewise, House 1 and House 4 on House 3, and House 2 and House 5 on 3, and House 2 and House 4 on 3, and House 4 and House 5 on 3, in each of these cases I found the conditional independence to hold by a reasonable test—not so severe as is conventional, but not too easy a test either.

“All right, and then what?”

“Then, Holmes, as before, I removed the edge in the diagram between each conditionally independent pair, with this result:



”What ho! Watson, it begins to look like the true plan of signs. But how in the world did you know the signals originated from House 1 and from House 2, rather from, say, House 4 or House 5 or House 3? Knowledge of the very direction of influence, which you claim, seems quite impossible unless you had extra information of some kind.”

“Nothing but the data you gave me Holmes. You will recall that in denuding my original full diagram—the one you complained about—I removed an edge between House 1 and House 2.

“Indeed.”

“You will further recall that in that case I removed the edge because House 1 and House 2 were independent in illumination. I manifestly did not have to consider whether House 1 and House 2 are independent conditional on House 3.

“Yes, I recall it.”

“In which case, House 1 and House 2 must both be causes of House 3 illumination.”

“Why so, Watson?”

“Pure logic, Holmes, your forte’, and using one of your principles: when the impossible is eliminated, what remains must be the case. Suppose I am wrong, Then Houses 1, 2, and 3 would have to be connected in one of three ways. Either

House 1 → House3 → House 2

or

House 1 ← House3 → House 2

or

House 1 ← House3 ← House 2

Were any of these cases correct, I should not have found the illuminations of House 1 and House 2 to be independent *simpliciter*, because there would be a causal connection between them, either from one to the other through House 3, or from House 3 to both House 1 and House 2. Were any of these three hypotheses true, I should have found House 1 and House 2 independent only conditional on the state of House 3. Since I found instead the simple independence between Houses 1 and 2, the only remaining possibility must be true: they both influence House 3.”

“Simply astonishing, old man, simply astonishing. And for the rest.”

“I leave you to work it out for yourself, Holmes, if you care to. By similar reasoning the illumination of House 3 must be the cause of illumination of both House 4 and House 5. I am done.”

“And my brain is exhausted. Tobacco ashes are so much simpler, Watson.”

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Search3 (PC Search)

File Logging Test Knowledge Layout

Parameters

Alpha: 0.11
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graph LR; X1 --> X3; X2 --> X3; X3 --> X4; X3 --> X5;
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Search5 (GES Search)

File Logging Knowledge Layout

Parameters

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Calc Stats

'Please note that some searches may take a long time to complete.

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graph LR; X1 --> X3; X2 --> X3; X3 --> X4; X3 --> X5;
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Save Cancel

8. Flowers for Moriarty

“I think I have got him, Watson. Using your methods, I have tracked Moriarty’s patterns. Each Tuesday, although sometimes Monday or Wednesday, a drayman delivers flowers to shops along High Street. A man wearing a bowler—a known associate of Moriarty’s buys a bouquet at the greengrocer and walks to the train station. There, he leaves the flowers in a stand outside a little shop, which sometimes sells flowers from the same place. A second man, always wearing a workman’s cap, takes the flowers from the stand and buys them again as if they had not already been purchased once. He then takes the flowers to a house in Maybury, although which house varies from occasion to occasion. I am convinced Moriarty must be the recipient, else there would be no need for so devious a procedure.”

“Fascinating, Holmes. So much trouble for a bit of brightness. Moriarty’s life must be very drab, his neck pinched from looking over his shoulder. Tell me, is this procedure invariable?”

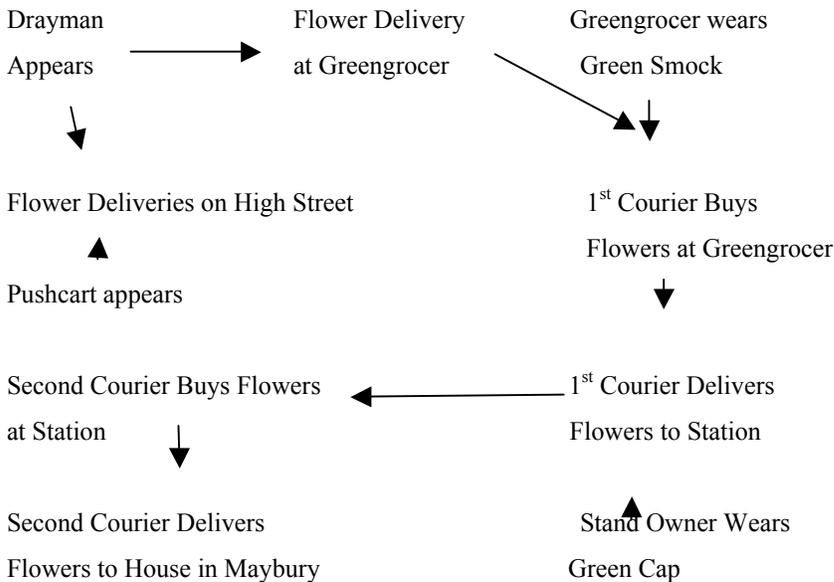
“No, it sometimes happens, as I said, that the drayman makes no deliveries on Tuesday. Now and then he misses a delivery in one place or another, including the greengrocer, the first courier sometimes does not appear even when there are flowers at the greengrocer, and similar uncertainties down the line. I have also noted that the flowers are seldom left at the train station unless the owner of the stand is wearing a green cap. Further, the first courier never purchases flowers the greengrocer is wearing a smock of the same color. Oh, and one last irrelevant bit: the flowers are sometimes delivered to High Street by a pushcart.”

“Does the pushcart fellow also deliver flowers to the greengrocer?”

“Never, so far as we can determine.”

“Perhaps, Holmes, you can diagram the thing for me.”

“I thought you might wish one of your pictures, Watson, so I have drawn it out already.” Holmes produced a sheet of paper on which he had written just so:



“Oh, this is excellent, Holmes. Am I to understand that the expression at the tail or head of each arrow stands for a condition that may be present or absent?”

“Indeed, Watson. The Irregulars have also collected data for me, as I thought you might want. I can tell you, for example, the probability that the 1st courier delivers flowers to the train station given that the stand owner is wearing a green cap that day and the courier has purchased the flowers at the greengrocer. The probability is quite high, indeed.”

“Wonderful, Holmes! I assume you have written out the probabilities as well.”

“Of course, Watson. Here they are, nicely arranged in the manner we were able to record them:”

$$\text{Prob}(\text{Pushcart Appears}) = .4$$

$$\text{Prob}(\text{Drayman Appears}) = .9$$

$$\text{Prob}(\text{Green Smock}) = .8$$

$$\text{Prob}(\text{Green Cap}) = .8$$

$$\text{Prob}(\text{Flower Deliveries on High Street given that Drayman and Pushcart Appear}) = .9$$

$\text{Prob}(\text{Flower Deliveries on High Street given that Drayman Does Not Appear \& Pushcart Appears}) = .7$
 $\text{Prob}(\text{Flower Deliveries on High Street given that Drayman Appears and Pushcart Does Not Appear}) = .8$
 $\text{Prob}(\text{Flower Deliveries on High Street given that Drayman Does Not Appear and Pushcart Does Not Appear}) = 0$
 $\text{Prob}(\text{Flower Deliveries at Greengrocer given that Drayman Appears}) = .8$
 $\text{Prob}(\text{Flower Purchase at Greengrocer given Flower Deliveries and Green Smock}) = .9$
 $\text{Prob}(\text{Flower Purchase at Greengrocer given otherwise}) = 0.$
 $\text{Prob}(\text{Flowers Left at Station given Flower Purchase and Green Cap}) = .95$
 $\text{Prob}(\text{Flowers Left at Station given Flower Purchase and no Green Cap}) = .2$
 $\text{Prob}(\text{Flowers Left at Station given otherwise}) = 0$
 $\text{Prob}(\text{2}^{\text{nd}} \text{ Courier Purchases Flowers given Flowers left at Station}) = .8$
 $\text{Prob}(\text{2}^{\text{nd}} \text{ Courier Purchases Flowers given otherwise}) = 0.$
 $\text{Prob}(\text{2}^{\text{nd}} \text{ Courier Delivers Flowers to Maybury given Purchases Flowers}) = .8$
 $\text{Prob}(\text{2}^{\text{nd}} \text{ Courier Delivers Flowers to Maybury given otherwise}) = 0.$

“Now, Holmes, what do you wish to do with this knowledge?”

“I have decided to apprehend the Greengrocer, Watson. I suspect, from his Greek looks, that the coinage with which the 1st courier pays contains a secret message. As you know, I have made a careful study of the ways in which messages can be encoded by the combination of ha’pence, pence, shillings and pounds used to sum up to any specified payment, and the possibility that foreign coinage may be used expands the possibilities most deliciously. ”

“If a message were to be passed to the Greengrocer, or from him, would it not be done more simply by an exchange of words, Holmes?”

“Ah, but words can be overheard, Watson.”

“Some of them. In any case, Holmes, why do you need my aid?”

“The problem is this, Watson. I wish to apprehend the Greengrocer this very morning, but only if the 1st courier has today made his purchase. Unfortunately, the lads were not all upon it today. The boy who was to watch for the drayman overslept, and the lad who was to watch at the train station has fallen sick at home, and none followed the 2nd courier—we do not know even if he arrived at the train station.”

“So why not wait until another day, Holmes?”

“I am impatient, Watson, and busy with other cases. If the matter can be unraveled today, then I would wish to have it done.”

“In that case, Holmes, I have one question: was there a boy observing whether the pushcart vendor appeared today?”

“There was, Watson, but nothing could be less relevant.”

“No harm then to tell me his report.”

“The pushcart did not appear today, Watson, since you insist.”

“Very well, Holmes, we must go for a stroll.”

“To where, Watson?”

“First down High Street, then to the greengrocer.”

“I cannot see the benefit of the exercise, Watson, when we have no evidence as to whether the courier appeared and purchased flowers. ”

“Trust me in this Holmes. We do have evidence, and it is a pleasant morning for a walk.”

.....

As we walked down High Street I stopped to purchase a rose for a boutonniere, while Holmes fairly stomped with impatience.”

“Confound it, Watson, when did you become such a fop? You are too old for it.”

“I know, Holmes, but the morning puts a spring in my step. Now to the Greengrocer’s. If the fellow is wearing a green smock, nab him.”

He was, and we did, not quite to Holmes’ satisfaction, but to mine. Unfortunately, all of the greengrocer’s coinage was British, but the man admitted a fellow in a bowler had purchased a sprig of flowers in the morning. To his disappointment, Holmes was unable to wedge out of him any further information as to Moriarty’s whereabouts, and the greengrocer was let go unhappily.

Walking back to his rooms, Holmes pummeled me with questions, and, I must say, not a little disdain for my judgment.

“How did you know, Watson, or were you merely lucky once more?”

“I did not *know*, Holmes, but it was about 70% likely on the evidence that the first courier had stopped and purchased flowers.”

“The evidence! Why Watson you insisted on knowing whether the pushcart vendor had sold flowers, which was obviously quite beside the point, you bought a boutonniere from sheer vanity, and you observed the greengrocer was wearing a green smock! If that was relevant, why not stop by the train station and see if the vendor there was wearing a green cap!? 70% be damned, Watson. You were merely lucky.”

“Not so, Holmes. I was following a quite definite and correct rule applied to the information that you provided me.”

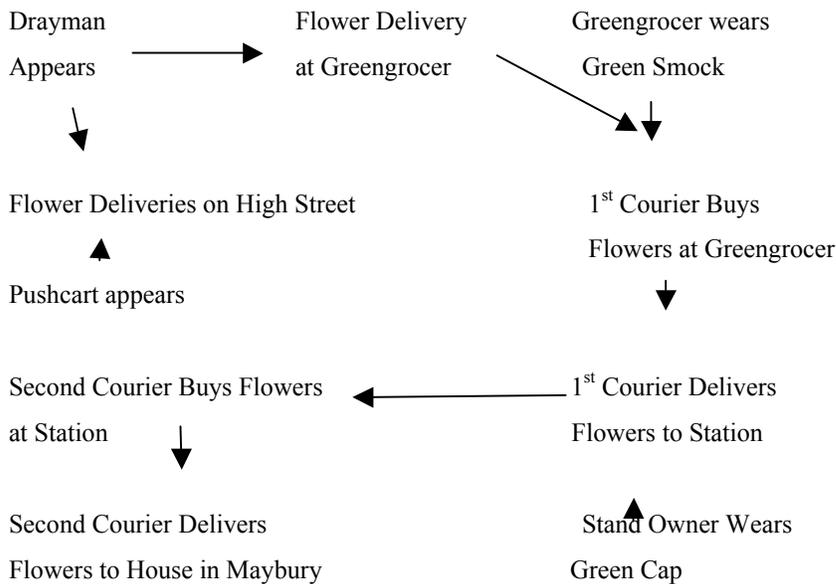
“What can that be, Watson?”

.....

“The explanation will require some patience, Holmes, so settle yourself with a good pipe and get your tobacco or a cigar from the coal shuttle. It may be a one-pipe problem, but it is a 2 pipe explanation.”

“Proceed.”

“Let us consider the diagram once more.”



“Let us focus on what is relevant information for predicting whether the 1st Courier buys flowers at the Greengrocer. I assume you agree that the color of the grocer’s smock is relevant evidence?”

“Certainly, if not green then no purchase.”

“Quite right, Holmes. And the flower delivery at the greengrocer?”

“Certainly again, for a similar reason.”

“Now

You are familiar with the idea that one variable or property is independent of another variable or property. For example, the probability that the greengrocer wears a green smock is independent of the probability that the station agent wears a green cap—when means the probability of both occurring equals in numerical value the number got by multiplying together the probability of each occurring.”

“Yes, I have that.”

“And so, you also have the idea that one variable or property is independent of another conditional on values of one or more other variables, no matter what the values of the latter may be?”

“Quite a mouthful, Watson.”

“Indeed, the idea is that if I know that flowers were purchased at the Greengrocers, then whether or not the greengrocer was wearing a green smock gives me no further information about whether the flowers were left at the train station. It is like so much clockwork, Holmes. A turn of the key winds the spring, and the spring turns the gears that turn the hands of the clock. Ordinarily, if I know the key has been turned, that provides me the information that the hands are probably turning. But if I remove the spring, or block it from compression, then whether the key has been turned or not gives me no information about the motion of the clock hands. Similarly, if I do not remove or block the spring, but merely observe its state of compression, knowledge as to whether the key has been turned or not gives no further information about the motion of the hands.”

“Very well, Watson. What next.”

“There is a correspondence between interventions that fix values for intermediate steps in a causal chain, on the one hand, and, on the other hand, what I call

conditioning on intermediate variables. Actions that block all of the causal pathways between two variables render them independent; so also they are independent among select cases in which there is a variable for each pathway that does not change in value. That is conditional independence of the cause and effect given the values of the intermediate variables. If the cause and effect are thus independent for all values of the selected intermediates, then I say the cause and effect variables are independent conditional on the intermediate variables.”

“Please, Watson, get to the point.”

“The point is that you know principles about causation and probability, Holmes. We have used them in previous cases.”

“What principles, Watson.”

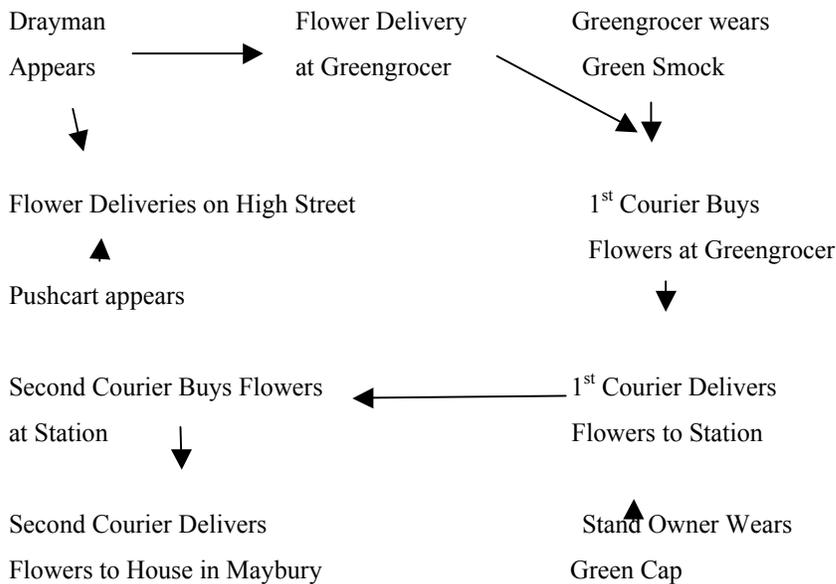
“These: First, a variable is associated with the variables it immediately affects. If you know the value of the variable affected, that provides information about the value of the causing variable. Second, consider any particular variable, call it X. Every variable Y that is not an effect of X, and is associated with X, must be associated by a causal chain from Y to X, or by some common cause of Y and X. Conditioning on all of the most immediate causes of X will therefore render Y independent of X—or put another way, Y and X will not be associated within the set of cases in which the direct causes of X are constant in value. Third, if a variable X has two causes, and they are themselves independent of one another, they will be associated, dependent, not independent, conditional on some values of X.”

“I had not realized we had employed so many principles in our cases, Watson. It seems so much simpler to apply them one by one as needed.”

“I agree, but to explain how one can reason about the flowers, all of the principles are required. So, to return to the matter of the flowers, let us suppose that for each kind of event in the diagram we can observe whether that event occurred, or not, and we wish to know the probability that the 1st courier buys flowers at the Greengrocer. What is the least number of features we can observe that is most informative for that question?”

“What do you mean, Watson?”

“In a different way, then, Holmes: what is the smallest collection of variables conditional on which all other variables are independent of whether or not the 1st courier buys flowers at the greengrocer?”



“Well, Watson, we must include whether the 1st courier delivers flowers to the station, for that is an effect, and we must include whether flowers are delivered to the greengrocer and whether he wears a green smock, for those are the direct causes in our diagram of whether the 1st courier buys flowers at the greengrocer. I think that is all.”

“You left out one necessary variable Holmes. We must also include whether the station agent wears a green cap, for that feature is a direct cause of whether the 1st courier delivers flowers to the station agent, which is a direct effect of the variable whose probability we have decided to estimate. By including the direct effect, delivery of flowers to the station agent, we have conditioned on a common effect of two variables, making them relevant to one another. Hence whether the station agent wears a green cap must be included in our set of variables.”

“Very clever, Watson. Is there a name for this minimal but maximally informative set of predictors of any given variable?”

“I have thought, Holmes, of calling it the Reichenbach Fall, after a philosopher who first came near the idea, since the probabilities fall off, as it were, on one side.”

A dark shadow passed over Holmes visage, as if he had foreseen some terror, and I thought it best to withdraw the proposal quickly.

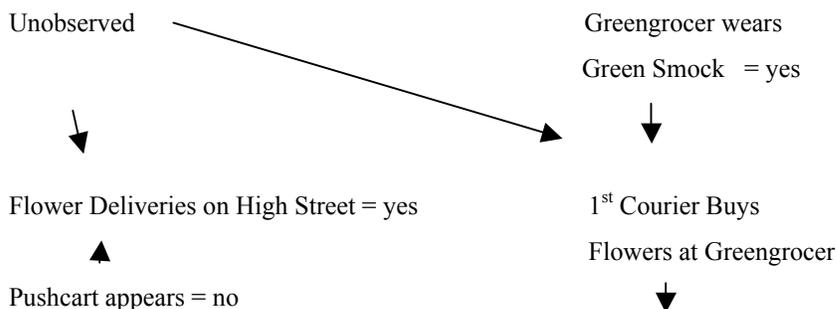
“Of course, one could think of the variables so selected as a set that marks off and blankets the target variable, so I have thought of calling it a Mark-Off Blanket, instead.

“If you do, Watson, the name will soon be Russified, so why not just call it the Markov Blanket?”

“Done, Holmes, the Markov Blanket it is.”

“But I still do not understand, Watson, why in our actual case, the appearance, or not, of the pushcart had the least relevance to whether the 1st courier had purchased flowers. It is not in the Markov Blanket in our diagram.”

“You are quite correct Holmes. It is not. But the diagram does not correctly describe the variables we measured. Recall that we did not know whether the drayman had delivered flowers on High Street, or to the Greengrocer. We did know that flowers had been delivered on High Street because I was able to purchase one. So the representation of our knowledge would be something like the following:



“Now, Holmes you should see that because we do not know whether the drayman appeared, we cannot put that fact into the Markov Blanket. Instead, we must put in whether the flowers were delivered on High Street. But there is a collider at flowers delivered at High Street and so we must also include in the Markov Blanket whether the pushcart appeared.

“So abstract, Watson.”

“But, Holmes, merely a diagrammatic way of making sure we do not confuse ourselves in applying common sense principles. After all, the reasoning is pure common sense: if there were flowers on High Street, and the pushcart did not deliver them, then the drayman must have, and so flowers were probably delivered to the greengrocer, and so, knowing that the greengrocer wore a green smock, we can compute the probability that the 1st courier purchased flowers.”

“Why not simply do that reasoning from the beginning, and eliminate the diagrams and principles, Watson?”

“Mechanizing reasoning as much as possible reduces the frequency of errors in reasoning, Holmes. Besides, one can in principle systematically find the Markov Blanket even in systems with thousands of variables. I would not want to trust to untutored judgement for that.”

“Watson, are you conning me? Do you claim that if you were provided with data for a thousand variables from land, sea and sky, or whatever, one of which was to be predicted from the others, you could find the Markov Blanket of that one?”

“In truth, I could not Holmes. I would weary or die before the calculations were done. But were Mr. Babbage a real engineer rather than a charlatan, it could be done.”

“In the meanwhile, Watson, I will continue to study tobacco, cloth and writing paper.”

9. The Case of the Intercepted Courier

“I need your help once more, Watson, but this time I am afraid, despite the somewhat mysterious reasoning you employ, the problem is impossible.”

“What can it be, Holmes?”

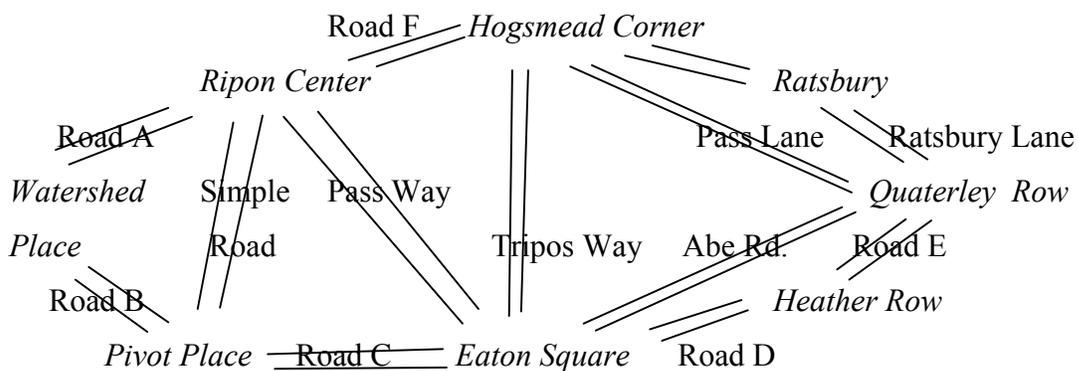
“We must intercept a courier making his way through London, and we must do so tonight.”

“What is so difficult about that?”

“The difficulty Watson, is that we do not know his route.”

“Perhaps you can explain a bit.”

“Enemy couriers have been passing from a house in Watershed Place to a house in Quaterley Row. Tonight, I have reason to believe a courier will travel with secrets that must be recaptured from him. There are a great many routes, more or less direct, between the two places, and I do not know which route he will take. I have had two teams of Irregulars out watching the couriers for many days and nights, and they have noted the appearance of each courier at selected places. That is all I know. The map looks like this:”



“Tell me Holmes, did the two teams watch for couriers in the same places?”

“They did not, Watson. Each team had a boy who could not stray too far from home. In consequence, while both teams watched Watershed Place for departures and Quaterley

Row for arrivals, one team also watched Eaton Square while the other watched Hogsmead Corner.”

“Tell me, Holmes, did the teams observe different sorts of couriers, women couriers by one team, for example, and men by the other?”

“Nothing of the sort, Watson. The times at which the teams went out to watch were effectively random.”

“So, Holmes, we have two sets of observations of the same system of couriers, but one set of observations has a variable not in the other set, and vice-versa.:

“That is the essence of the problem. It seems quite impossible, I know. ”

“And what do you want of me, Holmes?”

“We must intercept the courier, Watson, and there are only the two of us. Further, the couriers look to be ruffians according to the Irregulars—one the couriers pitched a rock at a boy, and nearly hit him at that—so we had best stick together. We cannot take the courier either at his point of departure or at his point of arrival—he has too many associates in Watershed Place, and it is too late if we take him in Quaterley Row. We must intercept him in between the start and the finish. But where? What route will he take?”

“Do you have the observations of the Irregulars, Holmes.”

“I do not. I gave them to Lestrade.”

“Blast, Holmes! Why did you do such a thing?”

“Lestrade wanted them for his calculator—his “statistician.” It seemed a reasonable request.”

“Do you know what the statistician found, if anything?”

“I know the words, but they make little sense to me. The statistician said that to discover anything about the routes we would have to “unify” the data for the two teams. He said that might be done by making up an observation of Hogsmead Corner for each observation of team 1, and making up an observation of Eaton Square for each observation of team 2, something he said we could do by “regression.” Imagine! He wanted to make up data! So, he calculated some regressions, but nothing came of it.”

“Did he inquire into conditional independencies, Holmes?”

“Yes, he said that in the data for team 1, the hypothesis that appearances of couriers at Watershed Place and at Quaterley Row are independent conditional on appearance at Eaton Square, could not be rejected. And he said from the data of team 2, the hypothesis that appearances at Watershed Place and at Quateley Row are independent conditional on appearance at Hogsmead Place likewise could not be rejected.”

“Well, at least he is good for something. Tell me one last thing, Holmes. Do the boys have any idea whether the couriers doubled back at any point, going along the same road twice?”

“According to their very rough estimates, there would not have been enough time. The couriers moved quickly between sightings There would not have been sufficient time even to have gone “backwards” from Eaton Square to Ripon Center/”

“Good. Get your coat, Holmes. I believe I know where we can intercept the courier.”

.....

“Where are we off to, Watson?” Holmes asked as they hailed a horse cab.

“To Tripos Way, my man, quick as you can.”

“Where in Tripos Way, sir? It be a long passage.”

“Anywhere. Whatever place is nearest.”

Holmes and Watson left the cab south of the intersection with Long Road, and waited.

“Which way do you think he will come, Watson? From the north, Hogsmead, or from the south, Eaton?”

“I have no idea, Holmes.”

“But Watson, in the case of the five signs your methods divined the direction of the signals. Surely you know which way the courier is going on the road.”

“I do not, Holmes. I only know that the courier will pass along this road, in one direction or the other, and even of that I am uncertain.”

“How uncertain?”

“As uncertain as the uncertainty in the judgements of Lestrade’s computer.”

But to their satisfaction, the courier did arrive, moving to the North. He came along with them without resistance.

.....

“All right, Watson, how did you do it this time?”

“Elementary, my dear Holmes. Because departure from Watershed Place is independent of arrival at Quaterley Row conditional on passing Hogsmead Place, the couriers’ routes most probably took them through Hogsmead Place. And because there is a similar independence conditional on Eaton Square, they likely also passed there. Hence they must have taken a route that connects Hogsmead Place and Eaton square. But every route that takes one from Watershed Place to Quaterley Row without doubling back must pass up or down Tripos Way.”

10. The Case of the Declining Bank Balances

“Watson, I have a suspicion.”

“Of what, Holmes?”

“That the bank is fiddling my account. I make deposits and withdrawals, and my account changes in consequence. The bank also deducts small fees and occasionally adds interest, which fluctuate unpredictably from months to month. The total change in the account is recorded at the end of each month. In between, my family trust always adds or subtracts exactly the opposite amount so that my account has 1,000 pounds. But it seems to me that over the last two years a good deal more money has disappeared from my account than ought to have done. I suspect a young man in the bank. He has a sly look and quick movements.”

“May I see your accounts then, Holmes? It sounds as if you have them for the last two years.”

“Indeed, I welcome your examination. You know I have no head for figures. Here is the account record”:

“Very interesting indeed, Holmes. Have you any one else’s accounts, for comparison?”

“Indeed, those of my brother Mycroft, who uses a different bank, in Geneva. He thinks he has a similar problem. My theory is that when I make a deposit or withdrawal, the man in my bank subtracts something from my account and somehow does the same in Mycroft’s. I suppose it is possible instead that someone in his bank is doing the misdeeds, but I quite distrust the young man I mentioned. I have noticed that he smokes Turkish cigarettes, quite expensive, and surely beyond his means.”

“But there is no way he could have access to Mycroft’s account, Holmes.”

”I suppose not. Still, he seems suspicious.”

		C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
1	MULT	SherAct	SherBal	MycroftBal	Mycroft											
1		-0.4099	999.4657	997.3287	-0.7485											
2		0.1147	999.3293	995.9011	1.5706											
3		-0.3495	999.3210	998.7428	-0.3285											
4		-1.2301	996.9703	998.9245	0.3731											
5		-0.6950	997.9134	997.5158	2.1569											
6		0.4362	1001.3069	999.8996	-0.1919											
7		-4.2946	998.4575	1004.1528	1.3465											
8		1.6542	998.5944	994.4348	2.1488											
9		-1.3606	996.9981	1001.3792	-1.3520											
10		0.8585	1003.3951	1000.9771	-1.4953											
11		0.3681	1001.2797	996.7068	2.1735											
12		-0.7563	1000.2122	1000.3993	0.0579											
13		-0.8519	999.0868	996.5266	2.8204											
14		-0.0238	998.0393	1000.2841	-1.0760											
15		-0.0625	1009.2004	1003.4750	4.9284											
16		0.8211	994.5018	996.2356	0.2546											
17		-0.4341	1000.9331	1001.7099	0.3208											
18		-0.9888	995.4882	993.2528	0.8220											
19		-0.4953	994.9149	999.9004	-1.1755											
20		0.1496	1001.5812	997.8783	1.7910											
21		-1.8512	999.0644	1003.0917	1.4112											
22		0.9685	1000.5490	996.7512	0.8836											
23		-0.4969	1000.4724	1003.2740	-1.4164											
24		-1.0323	1001.5251	1001.2891	0.3425											
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“Give me a moment, Holmes—actually, give me a few hours,” Watson muttered as he moved to a desk, took up a pen and began calculations.

.....

“I believe Holmes, that you and Mycroft must have a common solicitor.”

“We do, indeed, Watson, the firm of DC&H¹, who were solicitors to our father. But how did you know?”

“Because, Holmes, someone not at either bank has access both to your account and to Mycroft’s, and is stealing sums from both, monthly. A solicitor seemed the most likely.”

“Great Scot, Watson, do you mean it is not that low fellow at my bank? How can you be sure? Lestrade’s computer, the “statistician” as he calls himself, says it is quite

¹ Known more fully as Dewey, Cheatham and Howe.

impossible to infer causes that one has not observed, and I assure you I have not observed my solicitors steal from me.”

“No, Holmes, I cannot be sure, but I have excellent reasons provided by your bank records, and by Mycroft’s. If you demand an inquiry, I have some confidence that, although you may not recover your money, Lestrade will have a new customer, and soon enough Newgate prison will have a new resident.”

Holmes sent a message to Lestrade, requesting an inquiry without giving a reason, since he had none except Watson’s opinion. Lestrade reported the very next day that a DC&H solicitor found to be in financial straits had been arrested after records showed he had made a series of monthly withdrawals—and, oddly, occasionally deposits—from the accounts of Mycroft and Sherlock.

With the news, Sherlock summoned Watson to his rooms, and, carefully filling his pipe with shag and distributing a flame over the top of the bowl, asked:

“How on Earth did you know, Watson? Kindly dispense with numbers in your explanation.”

.....

“First, Holmes, I did not *know*—I might have been in error, but the chances were that I was not. Reasonable men and women infer the best explanation when it is far better than the next best.”

“Agreed, Lestrade’s computer seems always to be paralyzed by the possibility that the worst explanation is the true one. But get to the point; I am impatient.”

“Very well. You and Mycroft each made monthly deposits or withdrawals. The amounts the two of you deposited or withdrew were not associated in any way. Analysis of the data shows as much, but in any case I know your income is quite unpredictable.”

“True enough—our banking activities are not yoked to one another.”

“Further, Holmes, you will agree that your banking activities influenced your balance, and likewise those of Mycroft influenced his.”

“Of course, utterly banal.”

“But essential, Holmes. One can detect the consequent associations in the data you provided to me. Further any association between your brother’s banking activity and his balance is due to that influence, and similarly for you.”

“To be sure”

Watson moved to a table, and dipping his finger in some of Holmes’ cooled ash, and drew two diagrams:

Sherlock Activity → Sherlock Bank Balance
Mycroft Bank Balance ← Mycroft Activity

“I suppose, as usual, the arrows stand for causal connections, Watson?”

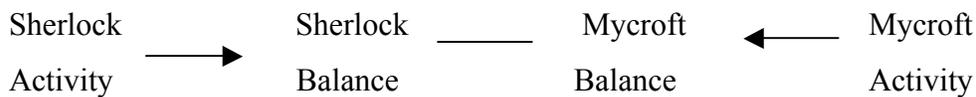
“Yes, Holmes.”

“Odd way to draw them--in opposite directions.”

“But to a point Holmes. As you know, to decide whether or not to include an arrow I must test the probability that there is no association, in this case by the correlation of the amount you deposit or withdraw and the amount of your balance, and similarly for Mycroft.”

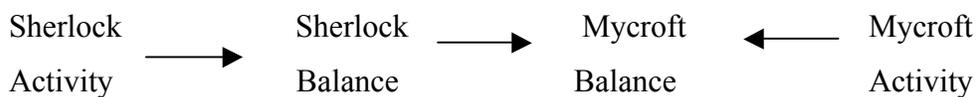
“Yes, confound it, get on with it and belay the mystery.”

“If I use a test only a bit weaker than that necessary to detect the associations given by the causal connections above, I also find that Mycroft’s bank balance is associated with Sherlock’s bank balance—and I find no other associations. Here look”

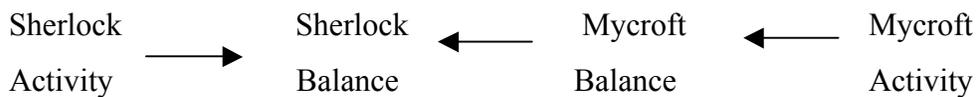


“So, Holmes, how am I to explain the association between your balance and Mycroft’s balance.”

“Why can it not be that my deposits are somehow influencing Mycroft’s balance, or his mine? Like this:”



“Or like this:”



“Because Holmes, in the former case, your banking activity would be correlated with Mycroft’s balance, which is not what the records evidence, and in the latter case, Mycroft’s activity would be correlated with your balance, which is also not evidenced in the data.”

“Watson, could it not be that my banking activity influences Mycroft’s balance, even though the data show no correlation?”

“That could be, Holmes, by sheer improbable chance, or because that influence is perfectly canceled by some path of influence between your banking activity and Mycroft’s balance.”

“Cartwheel, the new Professor down at that new university in London, seems to think we should not draw conclusions when such conspiratorial explanations are possible.”

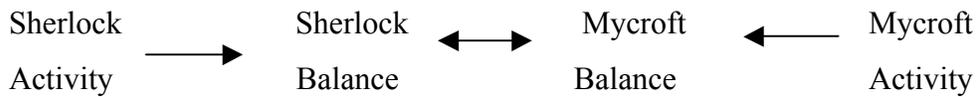
”Yes, Holmes, but conspiracy theories are always possible, and if they paralyzed inference, we should never conclude anything. She also thinks, by the way, that if you have lost your pipe in an evening walk through the park, you should not look for it under the lamps in the park.”

“Remarkable, why not?”

”Because, she holds that if one cannot search everywhere one should not search anywhere.”

“Well, Watson, we should give thanks that the police are only in league with the computers, and not with the Professor. Finish up, please.”

“As I was saying, the only explanation consistent with the facts—other than the bit of paranoia you mentioned, or else sheer chance--is that there must be an arrowhead at both ends of that directed line I drew. That is to say, the only explanation is an unobserved common cause of your bank balance and Mycroft’s. In confirmation, I find that Mycroft’s balance and your activity are associated, *conditional* on your balance, and, likewise, your balance and Mycroft’s activity are associated, *conditional* on his balance, which is just what we should expect.



“So it’s the old collider effect again, is it Watson.”

“Yes, it is, Holmes, the very thing Lestrade’s computer, Rubin, that “statistician”—such an ugly word—says never happens.”

“And finally, Holmes, I inferred that common cause must be the action of someone with access to both bank accounts, hence your common solicitor. A man in your bank is unlikely to have access to Mycroft’s account.”

“And why then did you offer your conclusion tentatively, Watson, with a hint of doubt and qualifications?”

“Because data are seldom perfect Holmes. To recover the association of your bank balance and Mycroft’s, I had to settle for a weaker test than that used to find the association of your deposits and your bank balance. So there was a possibility the association I found was due merely to chance.”

“Well, could you not have avoided all of these “tests”, as you call them, by using the methods of Thomas Bayes that you medical people now go on about.”

“No, Holmes, I could not have, unless I had a more fervent imagination. If I had hit on the idea of a common unmeasured cause directly, by myself, without attention to the associations, I could have used the late Reverend’s methods—Bayesian methods are splendid if you have an hypothesis in hand, but they are not quite so good at helping you find it, especially not when there may, for all one does not know, be unobserved factors at work.”

“And, finally, Watson, why were there sometimes additions to my balance, and to Mycroft’s, rather than subtractions? Did the thieving solicitor want to *give* me money?”

“He did, indeed, Holmes, sometimes, to keep away the suspicion that your account was being systematically looted.”

The screenshot displays a software application window titled "Data1 (Sim. M. Simulated Data)". The main window contains a table with the following data:

	MULT	C1	C2	C3	C4
1	1	-0.4099	999.4657	997.3387	-0.7485
2	1	0.1147	999.3293	995.9011	1.5706
3	1	-0.3495	999.3210	998.7428	-0.3285
4	1	-1.2301	996.9703	998.9245	0.3731
5	1	-0.6950	997.9134	997.5158	2.1569
6	1	0.4362	1001.3069	999.8996	-0.1919
7	1	-4.2946	998.4575	1004.1528	1.3465
8	1	1.6542	998.5944	994.4348	2.1488
9	1	-1.3606	996.9981	1001.3792	-1.3520
10	1	0.8595	1003.3951	1000.9771	-1.4953
11	1	0.3681	1001.2797	996.7068	2.1735
12	1	-0.7563	1000.2122	1000.3993	0.0579
13	1	-0.8519	999.0868	996.5266	2.8204
14	1	-0.0238	998.0393	1000.2841	-1.0760
15	1	-0.0625	1009.2004	1003.4750	-4.9284
16	1	0.8211	994.5018	996.2356	0.2546
17	1	-0.4341	1000.9331	1001.7099	0.3208
18	1	-0.9888	995.4882	993.2928	0.8220
19	1	-0.4953	994.9149	999.9004	-1.1755
20	1	0.1496	1001.5812	997.8783	1.7910
21	1	-1.8512	999.0644	1003.0917	1.4112
22	1	0.9885	1000.5490	996.7512	0.8836
23	1	-0.4969	1000.4724	1003.2740	-1.4164
24	1	-1.0323	1001.5251	1001.2891	0.3425
25					
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Overlaid on the main window is a "Search1 (PC)" dialog box. It features a "Parameters" section with "Alpha" set to 0.05 and "Depth" set to -1. Below these are "Execute*" and "Calc Stats" buttons. A "Result" section displays a "Result Pattern" diagram showing four nodes: X1, X3, X2, and X4. The nodes are connected by double-headed arrows: X1 ↔ X3, X3 ↔ X2, and X2 ↔ X4. A note at the bottom of the dialog reads: "Please note that some searches may take a long time to complete." At the bottom of the dialog are "Save" and "Cancel" buttons.

11. The Case of the Swiss Trusts

“Well, I have another for you, Watson, but this time no amount of “common sense,” as you call it, will solve the problem.”

“What is it Holmes.”

“A problem not entirely different from the question of Mycroft’s and my accounts, but on a larger scale. The Swiss government maintains two separate trust accounts. They place a portion of tax revenues in each at the beginning of the month, and on the next day they make a distribution from each to various cantons. The amount of distribution is in each case based on the balance in the trust.”

“Very good, Holmes, I understand. But what is the problem?”

“It is this. The Treasury Department official who wrote to me suspects that someone is making subtractions from both accounts on the day in between, but he is very unsure.”

“Then, Holmes, we can apply the same methods we applied to the matter of your bank account and Mycroft’s to see if there is an unrecorded common cause influencing the balances of the two trusts.”

“It is not so simple, Watson. The government keeps monthly records of tax receipt deposits in each trust, and a monthly record of the disbursements to each canton. They also keep monthly records of balances in each trust. Unfortunately the last are missing?”

“You mean there are no records at all of the balances?”

“Not quite, Watson. Apparently to prevent comparisons, someone has cut out the balance record for each odd month from the Geneva Trust, and cut out the balance record for each even month from the Basel Trust. So while we have records for each month of tax deposits and canton receipts to and from each trust, we do not have joint records of balances for the two trusts for any single month!”

“So the problem, Holmes, combines the difficulties of two of our previous problems: we must discover if there is an entirely unrecorded cause working on both of the trust balances, and we have in essence two data sets, one for even and another for odd months,

one data set with measures of the Basel balance, the other with measures of the Geneva balance, and no case, for no month, in which both are recorded.”

“That is the essence of the problem, Watson. I have of course asked for the remaining edges of the cut out pages, but they were cut with different instruments, one with a razor, one with a serrated knife, I believe of German steel.”

“Do you have the data, Holmes?”

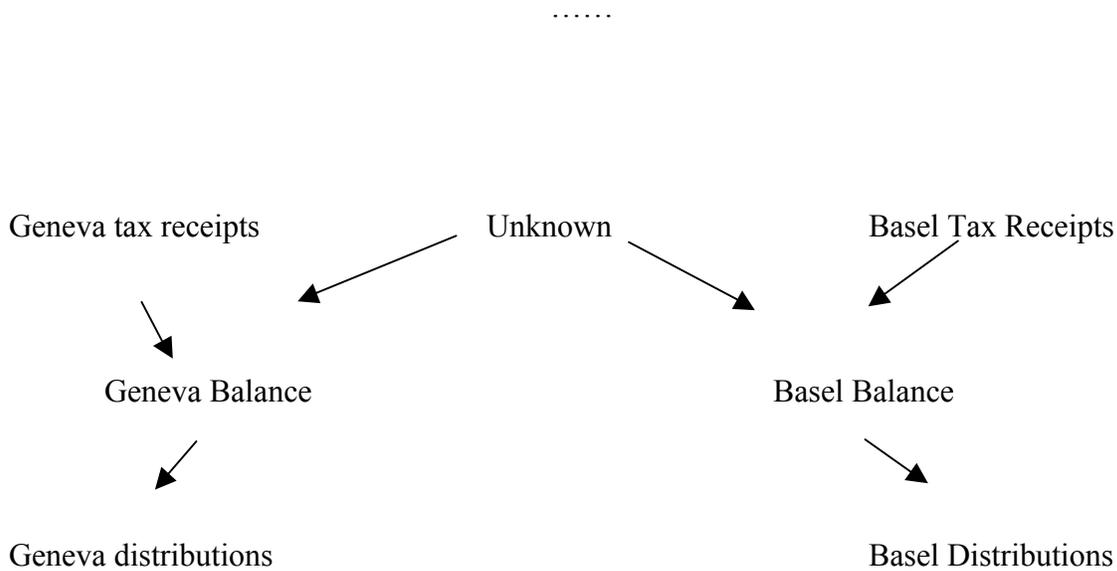
“I do, but I have something better. I took the problem to Lestrade’s “statistician.” He thought the problem intractable and wanted to do something he called “multiple imputation,” but I asked him to give me information you would want: which variables are independent of which others, conditional on which others.”

“And what did he report?”

“He said the information depends on something he called the “significance level” which he said he had given the value 0.05, meaning, he said, that 5% of the time he would decide wrongly that no independence holds. That seemed odd to me, but in any case here is he reports, Watson:

- Geneva tax receipts are independent of Basel tax receipts, and of Basel distributions.
- Basel tax receipts are independent of Geneva tax receipts and Geneva distributions.
- Geneva tax receipts are independent of Geneva distributions conditional on Geneva balance.
- Basel tax receipts are independent of Basel distributions, conditional on Basel balance.
- Basel distributions are independent of Geneva distributions, conditional on Basel balance.
- Basel distributions are independent of Geneva distributions, conditional on Geneva balance.

I took the sheet of paper from Holmes and studied it. Borrowing a sheet of paper, I drew a diagram:



“Assuming, Holmes, that Lestrade’s statistician has calculated correctly, here is my estimate of what is afoot, as you say. The diagram says it all.”

“As you say, Watson, but I do not see your reasoning.”

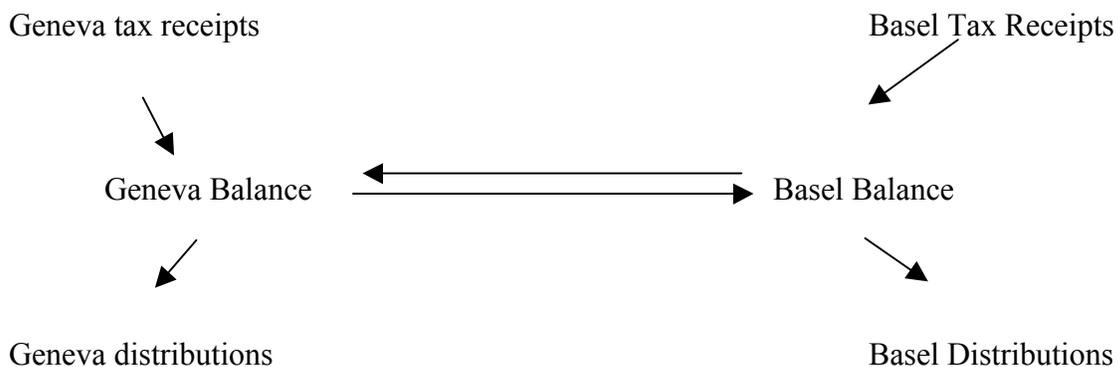
“Common sense, systematically applied, Holmes. The Geneva and Basel distributions are not independent, so there must be some causal connection that produces their association. Since, however, they are independent given Geneva balance, and likewise they are independent given Basel balance, and not independent given other variables, every causal connection between Geneva and Basel distributions must pass through both Geneva balance and Basel Balance. Hence there is a causal connection between Geneva Balance and Basel balance. It remains only to determine the direction of that causal connection or whether it comes from a common unrecorded cause. “

“Understood, Watson.”

“But further, since Geneva tax receipts and Basel distributions are independent, but are not independent given Geneva balance, Geneva balance must be a collider on some path between Geneva tax receipts and Basel distributions. Likewise, since Basel tax receipts and Geneva distributions are independent unconditionally, but are dependent conditional on Basel balance, Basel balance must be another collider. Hence the causal pathway

between the balances must be directed into both—but that is only possible if it is produced by a common cause, which in this case is not among the recorded variables. As the French say, *Voilà!*, Holmes.”

“Very clever, Watson, but this time you have made a mistake. There is another explanation.” Holmes drew the following diagram on my sheet of paper.



“Through some mechanism, Watson—the balances may each influence the other. Perhaps there is a secret transfer of funds back and forth between the two. That is perfectly consistent with your reasoning, but not with your conclusion.” Holmes almost grinned with satisfaction at my discomfiture.

“You are correct, Holmes, and my apologies. You have found another explanation for the facts I have noted and the reasoning I have given, but there is another fact in the data that I have neglected. Geneva tax receipts and Basel tax receipts are *not* independent given *both* Geneva balance and Basel balance. My hypothesis implies as much, for I have conditioned on two colliders, creating an association between the tax receipt variables. Your alternative explanation, to the contrary, requires such an independence, for in your hypothesis, if I condition on Geneva balance I block the pathway Geneva tax receipts -> Geneva Balance -> Basel Balance, and if I condition on Basel Balance I block the pathway, Basel tax receipts -> Basel balance -> Geneva balance. Conditioning on both, blocks all mechanisms of association between the two tax receipt variables.”

“Egad, Watson. Enough—it seems to me that you have conditioned on colliders in both.”

“I suppose that is enough, and I suppose that you will trust me that the rules of inference are somewhat less intuitive when we deal with systems in which variables have mutual inference. There are rules of inference nonetheless, and I have correctly applied them to your hypothesis.”

Final Musings

“How certain are your methods, Watson?” Holmes asked one evening as we smoked our pipes at ease in his rooms.

“I am certain neither of the applications nor of the principles. The applications depend on judgements from data, and those may always be in error. The principles are general and plausible, but sometimes false. They have various things to recommend them.

Mathematicians have proved that the principles we have used reduce to two: the principle we have called Markov, and one other, which is that, when applied to the true graph of causal relationships, the Markov principle captures all of the conditional and unconditional independence properties of the probability distribution of the variables.

When these two principles or assumptions hold, then, in the the long run, we are guaranteed to converge to correct causal information by the methods I have illustrated.

Indeed, the second principle can be somewhat weakened and that result obtained. But all guarantees about inferences are hypothetical—they depend on assumptions, principles.

My reasons for these are that we cannot reason about causation without the principle we used implicitly in defining Markov’s Blanket: each variable is independent of any set of variables it does not cause (directly or indirectly), conditional on its direct causes. We could not make inferences to the presence or absence of causation even from experiments such as Mr. Faraday’s without these principles. When the second of these principles fails, the data themselves will often tell us so, and warn us from making inferences to causes. The only alternative to them is skepticism or adhocery.”

“Adhocery?”

“Hypotheses made up in conformity with prejudices, according to no sound principles.”

“But Watson, what you say gives me no measure of what confidence I should have in conclusions when my data is finite—and my data is always finite.”

“We know of no way of calculating such measures for the results of genuine inquiry, Holmes, and so while I prefer principles that guarantee me success in the long run, they

give me no confidence in the long run, except this: if I use a method that is not correct in the long run then, using it, I can only be correct in the short run by luck.”

“But I have heard, Watson, that you are like the drunk who searches for his lost keys under the gas lamp because that is where the light is?”

“Ah, Holmes, would you have the drunk, or anyone, *not* search where the light is?”

“No, I suppose search should go on where the light is.”

“And would you have them waste effort searching in the dark?”

“Again, not.”

“I rest my case, Holmes.”