In Lecture 1, we have seen that Bayesian Networks can help us to intuitively represent and manipulate probability distributions over a large number of variables. This allows us to study more complex (i.e., realistic) situations, involving many variables.

What does this imply for formal epistemology? Well, many of the idealizations that are usually made can now be relaxed. Bayesianism is a case in point. The classical account only talks about a hypothesis (H) and a piece of evidence (E). In fact, however, scientific theories have an internal structure, there may be several pieces of evidence etc.

In this session, I present applications of Bayesian Networks to problems from the philosophy of science.

One does not have to be a Bayesian to apply Bayesian Networks! Clark Glymour is an example. He wrote the famous article “Why I am not a Bayesian”, but is one of the major forces behind the application of Bayesian Networks to Causal Inference.

Other contributors to this literature include P. Spirtes and R. Scheines (= Glymour’s collaborators at CMU), K. Korb, J. Pearl, J. Williamson, and J. Woodward.

The main idea is that a Bayesian Network is obtained that fit a given set of data. Under certain conditions, the edges in the network can then be interpreted causally.

We won’t focus on these applications and refer to the literature.
Bayesian Applications

- Starting point of Bayesianism: The psychologically plausible observation that we believe certain pieces of information with a certain degree (e.g. the report of the weather man that it’ll rain next week).
- Then: Argue that degrees of belief are probabilities (“Dutch Book Arguments”, Joyce, Leitgeb & Pettigrew).
- Central notion: Confirmation = probability raising
- Story: Start with a prior probability of a hypothesis H, i.e. \( P_0(H) \), learn evidence E, and update to obtain a posterior probability \( P_1(H) := P_0(H|E) \). (“Conditionalization”, “Bayes Rule”)
- Various extensions: other updating rules (such as Jeffrey Conditionalization), different measures of confirmation, ...

Bayesian Network Applications

Despite its virtues, the picture of science that Bayesianism presents is highly simplified.
Using Bayesian Networks, (at least) some of these idealizations can be relaxed. Here is a list of successful applications.

- What is a scientific theory?
- Confirmation of a hypothesis with several partially reliable instruments
- Confirmation transmission
- The Duhem-Quine Problem with probabilistic dependencies

Challenge: Develop Bayesianism, if possible, into a full-fledged epistemology of science!

My testing ground today: Intertheoretic reduction

Success Stories

Explanatory successes:
- Surprising evidence confirms better
- Variety of evidence thesis
- Duhem-Quine thesis
- Combining testimonies of different witnesses

Puzzles and concept explication:
- The raven paradox, the grue paradox
- Confirmation, coherence, and the coherence-truth link

Virtues:
- Bayesianism explains much with a few assumptions.
- It has a close relation to the sciences (e.g. causal discovery).
- It is a progressive (philosophical) research program.

Motivation

Reduction is one type of intertheoretic relation.
Why should one attempt a reduction?
- Metaphysical reasons
- Epistemological reasons

Three epistemological reasons:
- Explanation: The reducing theory explains (the success of) the reduced theory.
- Consistency: The two “stories” are made compatible.
- Confirmation: The conjunction of both theories is better confirmed after the reduction.

My goal: Study the relation between confirmation and reduction in a Bayesian framework.
Different parts of the academic community hold the following statements.

Claim 1 (Science): The aim of a reductive science is to derive the laws of the reduced science from more fundamental principles.

Claim 2 (Philosophy): Nagelian reduction is deeply flawed and has been overthrown decades ago.

Predicament: These two claims are contradictory!

→ So what shall we do?

Question: How does this work?

Reply: the declared aim is to deduce the laws of the non-fundamental theory from the laws of the fundamental theory.

Example: Statistical mechanics (SM).

Some quotes:

“The explanation of the complete science of thermodynamics in terms of the more abstract science of statistical mechanics is one of the greatest achievements of physics.” (Tolman 1938)

“The classical kinetic theory of gases is [a] case in which thermodynamics can be derived nearly from first principles.” (Huang 1963)


Colloquially: Gases spread, coffee cools down, etc.
Mechanics: The gas is a collection of molecules. These are governed by the laws of mechanics. So in principle everything that there is to know about the gas follows from the laws of mechanics.

Question: how do these two ways of describing the gas go together?

Statistical Mechanics: Derive the laws of TD from Mechanics plus some probabilistic assumptions. What exactly does that mean?

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Example 1 (Equilibrium): The Ideal Gas Law

TD: gives us the IGL: \( pV = kT \)

SM: Describe state of the gas in mechanical state space, and it postulates a velocity distribution \( f(v) \) telling us how probable it is that a randomly chosen molecule has velocity \( v \).

Assume the following relations that connect magnitudes in TD and in SM:

1. Pressure: \( p = F/A \)
2. Temperature: \( T = \frac{2n}{3k} \cdot E_{\text{kin}} \)
3. Volume: \( V = V \)

Some straightforward calculations then show that:

**Premise 1**: Mechanical description of the gas

**Premise 2**: Velocity distribution \( f(v) \)

**Premise 3**: \( p = F/A, T = \frac{2n}{3k} \cdot E_{\text{kin}}, V = V \)

**Conclusion**: \( p \cdot V = k \cdot T \)

Upshot: We have derived IGL from the laws of SM.

And notice: this is not a "philosophical reconstruction"; this is exactly how it is done in textbooks!

Example 2 (Non-Equilibrium): The Second Law of TD

The Second Law of TD says that the TD entropy \( S_{TD} \) cannot decrease in an isolated system (and that it typically will increase).

So from a TD perspective the spreading of the gas is described by an increase in entropy.

Aim of SM: derive this law from the principles of SM.

Notice: This is really what the aim of the foundation of SM is! There are hundreds of papers written on this topic.
The Project

So what those working on the foundations of TD and SM would like to have is an argument of the following kind:

Premise 1: The principles of SM
Premise 2: Some auxiliary assumptions (possibly)

Conclusion: The Second Law of TD

Note: All practitioners in the field as well as the philosophers of physics working on SM agree that this is the aim – there is no controversy about this.

2. Philosophy

What notion of reduction is at work here?
Deducing the laws of one theory from another one rings a bell: this is Nagel’s account of reduction!

Premise 1: Laws of fundamental theory $T_F$
Premise 2: Auxiliary assumptions

Conclusion: Laws of phenomenological theory $T_P$

(Details and qualifications to follow.)

An Awkward Situation

- Salient Point: This squares perfectly with what happens in TD and SM!
- So Nagelian reduction seems to be the (usually unacknowledged) background philosophy of SM!
- So philosophy and scientific practice seem to live in harmony . . . or so it seems.
- The problem is that there is a consensus in the philosophy community these days that Nagelian reduction is wrongheaded and outdated!
- So we are in an awkward situation: What seems to be conceptual backbone not only of SM, but also of quantum chemistry, wave optics, etc. is considered to be philosophically unacceptable!

Resolving the Dilemma

Option 1: Reject what scientists are doing → No.
Option 2: Re-analyse the scientific practice in different terms. → Yes . . . but what terms?

Plan:
- Have a look at the criticisms of Nagel’s theory.
- Defend the Generalized Nagel-Schaffner Model of Reduction (GNS).
- Re-examine the physics examples in the light of GNS.
- Explore, on the basis of GNS, when a purported reduction is justified epistemically.

No.s 1 to 3 have been addressed in a companion paper. Here we focus on no. 4.
As we have seen, the basic idea is:

Premise 1: Laws of fundamental theory \( T_F \)
Premise 2: Auxiliary assumptions

\[ \text{Conclusion: Laws of the phenomenological theory } T_P \]

Two cases

(a) Homogeneous reductions: both theories use the same terms. Example: Kepler’s laws and Newton’s theory.

(b) Inhomogeneous reductions: The theories don’t use the same vocabulary. Example: TD and SM

In the latter case we need so called bridge laws to connect the two vocabularies.
Examples: \( p = F/A \) and \( T = 2\pi n/3k \cdot E_{\text{kin}} \)

One Criticism

Nagelian reduction is de facto unrealisable because exact derivability can almost never be achieved (cf. the Second Law).

Reply: True, but Schaffner (and in fact Nagel himself) have addressed this point by revising the original proposal.

Historical note: this happened in the 1970 – hence before NWR even came into existence.
**A Survey**

Intertheoretic Reduction I: Non-Formal

Intertheoretic Reduction II: Formal

Open Problems

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The Generalized Nagel-Schaffner Model of Reduction

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Example

It turns out to be impossible to derive the exact Second Law of TD from SM. But we don’t need this (Callender 1999): TD is confirmed only on scales (both in size and time) where SM fluctuations are unobservable. So there is no real clash.

If we can derive that the entropy goes up and then only fluctuates a bit (call this Second Law∗) then that’s all we need.

Second Law and Second Law∗ are then strongly analogous.

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Formalizing GNS

Three Steps

1. Adopt auxiliary assumptions describing the particular setup under investigation. Then derive from these and $T_F$ a restricted version of each element $T_{F}^{(i)}$ of $T_F$. Denote these by $T_{F}^{(i)}$ and the corresponding set by $T_{F}^{*} := \{T_{F}^{(1)}, \ldots, T_{F}^{(n_F)}\}$.

2. $T_F$ and $T_P$ are formulated in different vocabularies. Substituting the terms in $T_{P}^{*}$ with terms from the macro theory as per the bridge laws yields $T_{P}^{*}$, i.e. the set $\{T_{P}^{*(1)}, \ldots, T_{P}^{*(n_P)}\}$.

3. Show that each element of $T_{P}^{*}$ is strongly analogous to the corresponding element in $T_{P}$.

N.B. This is a reconstruction. In practice, scientists might work both ways.

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Evidence

Three Kinds of Evidence

1. Evidence that only confirms, to some degree, the phenomenological theory. Example: The Joule-Thomson process.

2. Evidence that only confirms, to some degree, the fundamental theory. Example: the temperature-dependence of a metal’s electrical conductivity.

3. Evidence that confirms, to some degree, both. Example: consider again the gas confined to the left half of the box which spreads evenly when the dividing wall is removed. It follows from TD that the thermodynamic entropy of the gas increases; at the same time, it is a consequence of SM that the Boltzmann entropy increases in that process. So the spreading of the gas confirms both SM and TD.

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Before the Reduction

We assume that $T_F$ and $T_P$ have only one element, viz. $T_F$ and $T_P$ respectively. Furthermore, $E$ confirms $T_F$ and $T_P$, $E_F$ only confirms $T_F$ and $E_P$ only confirms $T_P$.

Let the probability measure before the reduction be $P_1$.

Fixing the Probabilities

To complement the Bayesian Network, we specify the following values:

$$P_1(T_F) = t_F, \quad P_1(T_P) = t_P$$
$$P_1(E_F|T_F) = p_F, \quad P_1(E_F|\neg T_F) = q_F$$
$$P_1(E_P|T_P) = p_P, \quad P_1(E_P|\neg T_P) = q_P$$
$$P_1(E|T_F, T_P) = \alpha, \quad P_1(E|T_F, \neg T_P) = \beta$$
$$P_1(E|\neg T_F, T_P) = \gamma, \quad P_1(E|\neg T_F, \neg T_P) = \delta$$

Results

1. $E_F$ and $T_F$ are probabilistically independent:
   $$P_1(T_F|E_F) = P_1(T_F)$$

2. $E_F$ and $T_P$ are probabilistically independent:
   $$P_1(T_P|E_F) = P_1(T_P)$$

3. $T_F$ and $T_P$ are probabilistically independent:
   $$P_1(T_F, T_P) = P_1(T_F) P_1(T_P)$$

After the Reduction

Recall our three steps: (i) Derive $T_F^*$ from $T_F$ and the auxiliary assumptions. (i) Introduce bridge laws and obtain $T_P^*$ from $T_F^*$. (iii) Show that $T_P^*$ is strongly analogous to $T_P$.

Note that there is now a direct sequence of arrows from $T_F$ to $T_P$, i.e. the path through $T_F^*$ and $T_P^*$. 

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After the Reduction

Fixing the Probabilities

For any $T_F$, let

\[ P_2(T_F|T_P) =: p_F, \quad P_2(T_F\neg T_P) =: q_F \]

The bridge law amounts to this:

\[ P_2(T_F|T_P^*) = 1, \quad P_2(T_F\neg T_P^*) = 0 \]

All other probability assignments hold as in the case of $P_1$.

Three Remarks

1. $T_P^*$ may be more or less good. How good it is depends on the context (i.e. the application in question and the auxiliary assumptions made) and on the judgement of the scientists involved. In line with our Bayesian approach, we assume that the judgement of the scientists can be expressed in probabilistic terms.

2. The move from $T_F$ to $T_P$ in virtue of the bridge laws may well be controversial amongst scientists. Whilst bridge laws are non-conventional factual claims, different scientists may assign different credences to them.

3. What counts as strongly analogous will also depend on the specific context and on the judgement of the scientists. For example, whether entropy fluctuations can be neglected or not cannot be decided independently of the specific problem at hand, see Callender (2001).

Confirmation Flow

Confirmation Flow 1: When does $E_F$ confirm $T_P$?

**Theorem 1**

$E_F$ confirms $T_P$ iff $\left( p_F - q_F \right) \left( p_F^* - q_F^* \right) \left( p_F - q_F^* \right) > 0$.

Hence $E_F$ confirms $T_P$ if: (i) $E_F$ confirms $T_F$ (i.e. $p_F > q_F$), (ii) $T_F$ confirms $T_P^*$ (i.e. $p_F > q_F^*$), and (iii) $T_F$ confirms $T_P$ (i.e. $p_F > q_F^*$).

Confirmation Flow 2: When does $E_P$ confirm $T_F$?

**Theorem 2**

$E_P$ confirms $T_F$ iff $\left( p_P - q_P \right) \left( p_P^* - q_P^* \right) \left( p_P - q_P^* \right) > 0$.

Hence $E_P$ confirms $T_F$ if: (i) $E_P$ confirms $T_P$ (i.e. $p_P > q_P$), (ii) $T_F$ confirms $T_P^*$ (i.e. $p_F^* > q_F^*$), and (iii) $T_F$ confirms $T_P$ (i.e. $p_F^* > q_F^*$).
We calculate the difference,
\[ \Delta := P_2(T_F, T_P|E, E_F, E_P) - P_1(T_F, T_P|E, E_F, E_P) \]
and obtain
\[ \Delta = (p_F^* - q_F^*)(p_P^* - q_P^*) t_F \alpha \tilde{\Delta}. \]

(\(\tilde{\Delta}\) is given in the paper.)

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**The Posterior Probability of the Conjunction (cont’d)**

**Theorem 4**
\[ \Delta = 0 \text{ if } (p_F^* = q_F^*) \text{ or } (p_P^* = q_P^*). \]

- Interpretation: If either \(T_F\) and \(T_P\) are independent, then the flow of confirmation from \(T_F\) to \(T_P\) (and vice versa) is stopped and the epistemic situations before and after the ‘reduction’ are the same.

- Further result: The posterior probability of the conjunction of \(T_F\) and \(T_P\) increases after a reductive relationship is established between the two theories.

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**Upshot**

1. GNS reduction between two theories, such as TD and SM, is epistemically advantageous in virtue of the results mentioned above (and further results presented in the paper).
2. A reduction makes sure that evidence which, prior to reduction, only supported one of the theories, due to the reduction comes to support the other theory as well.
3. Successful reduction increases the posterior probability of the conjunction of both theories.
4. True, but not shown here: Reduction also increases the prior probability of both theories and, in many cases, results in the conjunction of both theories being better confirmed by the total evidence.

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**My Methodological/Philosophical Approach**

- I adopt a **scientific approach** to philosophical questions and construct models in the framework of a theory (in this case: Bayesianism). This is very much analogous to what is going on in, say, Newtonian Mechanics.
- I want to explore how far Bayesianism can be pushed as a candidate for a general framework for the epistemology of science.
- And so I would like to extend the scope of Bayesianism much beyond confirmation theory.
- The philosophical assumption behind all this is a thoroughgoing empiricism. Science is in the first place about the relation between data and theories, and all other notions derive from this, so I tentatively assume.
Open Problems

1. Study other types of intertheoretic relations. This requires a combination of philosophical methods. Generalizations from case studies help identifying types, and Bayesian modeling provides an epistemological analysis. Current project: Montague grammar.

2. Provide a measure of explanatory power.

3. Apply these insights to the debate about inference to the best explanation.

4. Provide a Bayesian measure of simplicity.

5. . . .

References

The session is based on joint work with Foad Dizadji-Bahmani (LSE) and Roman Frigg (LSE). It contains material from the following two papers:


Both papers are downloadable from the PhilSci Archive (http://philsci-archive.pitt.edu/).