Bayesian Networks in Epistemology and Philosophy of Science
Exercise Set 1

1. A dice is thrown. Let A be the proposition "We observe an even number." and B the proposition "We observe a number ≤ 4." Are A and B independent?

2. (a) Suppose that the rate of disease D is three times higher among homosexuals than among heterosexuals, that is, the percentage of homosexuals who have D is three times the percentage of heterosexuals who have it. Suppose, further, that Pat is diagnosed with the disease, and this is all that you know about Pat. In particular, you don't know anything else about Pat's sexual orientation; in fact, you don't even know whether Pat is male or female. What is the probability that Pat is homosexual?

   (b) A small company has just bought three software packages to solve an accounting problem. They are called Fog, Golem, and Hotspot. On first trials, Fog crashes 10% of the time, Golem 20% of the time, and Hotspot 30% of the time. Of ten employees, six are assigned Fog, three are assigned Golem, and one is assigned Hotspot. Luca was assigned a program at random. It crashed on the first trial. What is the probability that he was assigned Hotspot?

   (c) How is this exercise related to the so-call *base rate fallacy*, well-known from the work of Tversky and Kahneman?

3. Proof the Chain Rule (whenever $P(\mathrm{A}_1, \mathrm{A}_2, \ldots, \mathrm{A}_\mathrm{n}) \neq 0$):

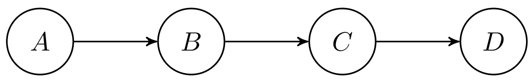
$$P(\mathrm{A}_1, \mathrm{A}_2, \ldots, \mathrm{A}_\mathrm{n}) = P(\mathrm{A}_1|\mathrm{A}_2, \ldots, \mathrm{A}_\mathrm{n}) \cdot P(\mathrm{A}_2|\mathrm{A}_3, \ldots, \mathrm{A}_\mathrm{n}) \cdots P(\mathrm{A}_\mathrm{n})$$

4. Show that if A is unconditionally independent of B, then A is also unconditionally independent of ¬B, and ¬A is unconditionally independent of ¬B. Does this pattern generalize to conditional independencies?

5. Show that the four Semi-Graphoid Axioms hold if $A \perp\!\!\!\perp B|C \Leftrightarrow P(\mathrm{A}|\mathrm{B},\mathrm{C}) = P(\mathrm{A}|\mathrm{C})$.

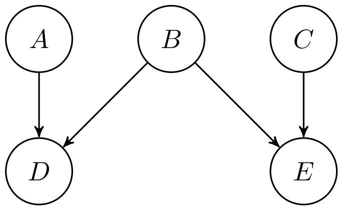
6. Proof the Product Rule for Bayesian Networks, i.e. $P(A_1, \ldots, A_n) = \prod_{i=1}^{n} P(A_i|par(A_i))$.

7. Calculate $P(\mathrm{B}|\mathrm{A},\mathrm{D})$ for a Bayesian Network depicted below and $P(\mathrm{A}) = a$, $P(\mathrm{B}|\mathrm{A}) = p_B$, $P(\mathrm{B}|\neg\mathrm{A}) = q_B$ etc.



8. According to PMC, $D \perp\!\!\!\perp A|C$ for the Bayesian Network depicted above. It is intuitive that also $D \perp\!\!\!\perp A|B$. Use the Semi-Graphoid Axioms to find out.

9. Use PMC to determine conditional independencies that hold in the Bayesian Network below.



10. Use the $d-$separation criterion to determine several conditional independencies that hold in the Bayesian Networks below and that do not follow directly from PMC.