Between Unity and Disunity: A Bayesian Account of Intertheoretic Relations

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Motivation

- How *is* science organized? Does it form a unified whole, or does it cluster in many more or less unconnected patches? – This is a descriptive question.
- How *should* science be organized? How would we like it to be, and why? – This is a normative question.
- Both questions are relevant for a wide range of issues from metaphysics (do we live in a dappled world?) to public policy (who should get funding?).
- This talk will examine two extreme positions – the unity of science and the disunity of science programs – and defend a middle ground between the too.
- To make my claim I will combine the *case study method* with *philosophical modeling*.

The Unity of Science

- Driving intuition: One important goal of scientific theorizing is to come up with a single unified theory ("simplicity").
- Two well-known proposals:
  1. Oppenheim and Putnam's (1958): pyramid structure of the sciences, ontological and epistemological reduction, unification as a heuristics for theory construction
  2. Nagel (1961): model of reduction employing biconditional bridge laws and subsequent deduction of the reduced to the reducing theory
- Unity on all levels: sciences, theories etc.

. . . and Its Problems

- Both proposals focus on *deductive relations* between theories. This turns out to be too strong a requirement, and so it is no surprise that . . .
- . . . no real cases fit these models.

However:

- Disregarding the main intuition behind the unity-of-science program altogether is like throwing out the baby with the bath water.
- For example, there might be epistemically relevant *inductive* relations between theories.

The Disunity of Science

- Driving intuition: There is no hierarchy of theories (and sciences), "all theories are equal", and all of them are indispensable.
- Supported by case studies from "messy" science (lasers, superconductivity, . . .). We find many theoretical approaches, none of them is fully satisfying, but all of them illuminate some aspect of the phenomenon in question while unified accounts often fail to do so ("strength").
- Cartwright: "Theories and models are only connected as they relate to the same empirical reality." There is no deeper relation between them beyond that.
- Disunity on all levels: sciences, theories etc.

. . . and its problems

- Cartwright's claim is too extreme as it disregards the evident interrelatedness (and mutual support) of theories (models, and whole sciences) as well as scientists' (often successful) ambition to come up with unified accounts.
- No argument is given why this interrelatedness is not epistemically relevant. It should not be neglected.

Upshot

- While adherents of the unity of science idea stress the goal to present a single theory ("simplicity") that accounts for everything (but often fails to account for much), disunifiers stress (or build on) our goal to account for as many empirical phenomena as possible ("strength").
- While some unified theories do well on both counts (e.g. Maxwell's theory), there is typically a *tradeoff* between the two goals. They pull in different directions.

My Goal for the Talk

- Construct an alternative *general* philosophical account that is descriptively adequate and normatively interesting.
- More specifically, I will defend the following claims:
  1. Descriptive claim: Science is organized, at all levels, as an increasingly coherent network of the respective units of analysis (disciplines within science, theories within a discipline, models within a theory) ⇒ case study
  2. Normative claim: Coherent networks provide the best tradeoff between simplicity and strength. ⇒ philosophical modeling

Hadron Physics

- Hadron physics studies the static and dynamic features of strongly interacting particles (such as nuclei, protons, neutrons, pions, quarks and gluons).
- The field is very diverse and we find a whole range of theoretical accounts.

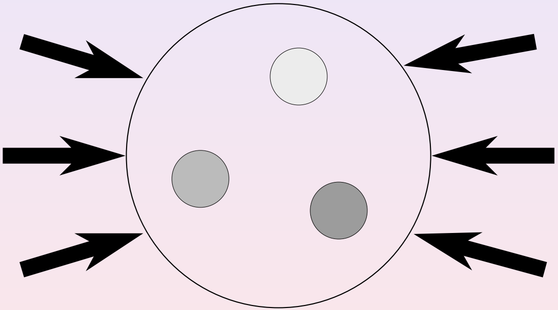
The Theory: QCD

- Quantum chromodynamics (QCD) is considered to be the fundamental theory of strong interactions.
- However, analytical solutions can only be obtained for high energies.
- In the low-energy domain (i.e. the domain of many nuclear physics applications), theoretical progress is hard to come by.
- Much work is based on computer simulations that, however, do not provide much, if any, understanding and still fail to account for a larger number of phenomena.

Models

- Models, such as the MIT bag model, are much easier to solve, visualizable, and provide us with an understanding of what is going on inside a hadron.
- Models also help scientists to identify the relevant degrees of freedom.
- QCD and these models are only loosely related. There is no (not even approximate) deductive relation between theory and model, and sometimes there is only a story one can tell to make the model plausible.

The MIT Bag Model



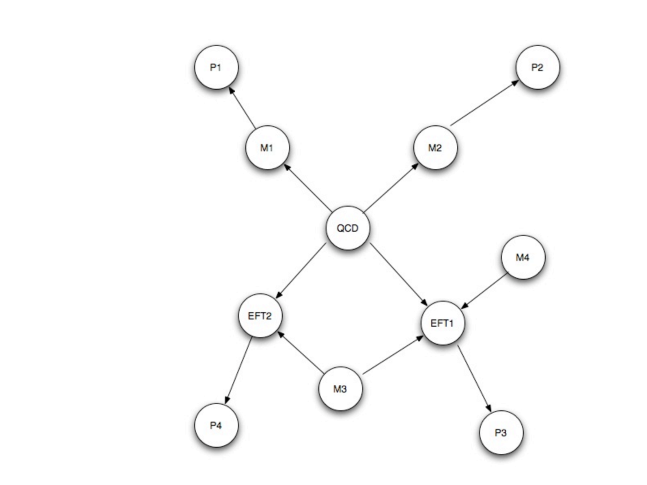
Effective Field Theories (EFTs)

- EFTs are an important theoretical tool for nuclear and particle physicists.
- They can be (approximately) derived from the fundamental theory (i.e. QCD) and account for the physics at a given energy scale.
- EFTs are non-renormalizable.
- According to EFTs, the world appears to be a layered structure of quasi-autonomous domains, each with its own ontology.

Interrelatedness

- If unification is the goal, why not just working with QCD?
- Note that, for practical reasons, QCD does not account for many low-energy phenomena while models and EFTs do.
- Note further that there are interesting relations between QCD and the models and EFTs.
- Theory, models and EFTs form an interrelated ("coherent") whole and become more and more coherent in the course of science. Each of them serves a specific *function* in the research process and is indispensable for scientific theorizing.
- There is a tradeoff between the goal of having a general and unified theory on the one hand (QCD is much more general than an EFT), and the goal to account for as many empirical phenomena as possible on the other hand (models and EFT are much better than QCD in this respect).

Summing Up: The Structure of Hadron Physics



What Do We Conclude from This?

- Remember my hypothesis: Coherent networks of disciplines, theories, models, . . . , achieve the best tradeoff between simplicity and strength.
- So far we have established that one subfield of physics exhibits this structure.
- But:
  1. Is hadron physics typical? $\Rightarrow$ now
  2. Can the hypothesis by justified normatively? $\Rightarrow$ coming soon

Are Coherent Networks Typical in Science?

One sees coherent networks on all levels:

- There are important methodological (e.g. simulations), epistemic and, perhaps, metaphysical relations between different sciences.
- The emergence of new sciences (e.g. nanoscience, quantum information etc.) and new subfields (e.g. neuroeconomics) make the whole of science more coherent.
- Some theories (e.g. evolutionary theory) play an integrating role in their discipline.
- . . .
- Remember, however, the story that Patrick Suppes tells in his *Probabilistic Metaphysics*.

Motivating Bayesianism

- How can the normative claim be established?
- To do so, we have to go beyond the descriptive ("naturalistic") framework adopted so far.
- I propose to adopt a normative philosophical framework that is flexible enough to allow for empirical input and to address our main question.
- As a philosophical framework, I choose *Bayesianism*.

A Primer on Bayesianism

- Quantitative confirmation theory: When (and how much) does a piece of evidence E confirm a hypothesis H?
- Typically formulated in terms of (subjective) probabilities
- Normative theory (see Dutch books)
- Confirmation = positive relevance between H and E
  - Start with a prior probability $P(H)$
  - Standard updating-rule ("learning"): $P_{new}(H) := P(H|E)$
  - By mathematics ("Bayes Theorem") we get:

$$P_{new}(H) = \frac{P(E|H)P(H)}{P(E)}$$

  - H confirms E iff $P_{new}(H) > P(H)$
  - H disconfirms E iff $P_{new}(H) < P(H)$

Extending the Scope of Bayesianism

- Bayesianism makes many idealizations which can be relaxed (e.g. by using the theory of Bayesian Networks).
- Bayesianism can be seen as a modeling framework that allows the inclusion of empirical input (*Naturalized Bayesianism*).
- Traditional Bayesianism makes a clear distinction between a hypothesis and evidence. Even if this distinction is only pragmatic, it becomes dubious in the light of theory-ladeness. So I propose to treat theories, models, phenomena all at the same level and simply calculate the joint probability of all those.
- Amongst alternatives, we accept the configuration with the highest probability.

A Bayesian Analysis

- Goal: Give a Bayesian account of the unity-of-science debate.
- Make plausible that coherent networks do best in terms of probability maximization.

The Unity of Science from a Bayesian Point of View

- Highly unified theories are often too complicated to apply. They only account for a few phenomena.

$$\begin{aligned} P(T, E_1, \ldots, E_n) &= P(T, E_1, E_2) \cdot P(E_3, \ldots, E_n) \\ &= P(T, E_1, E_2) \cdot \prod_{i=3}^{n} P(E_i) \\ &\approx 0 \end{aligned}$$

The Disunity of Science from a Bayesian Point of View

- Highly disunified theories also have a very small joint probability:

$$P(T_1, \ldots, T_n, E_1, \ldots, E_n) = \prod_{i=1}^{n} P(T_i, E_i) \approx 0$$

Coherent Networks of Theories and Models

- Interrelated networks can have a sufficiently high probability.
- The joint probability $P(T_1, \ldots, T_k, M_1, \ldots, M_l, E_1, \ldots, E_m)$ can be written as a product

$$P(T_1 | T_2, \ldots, E_m) \cdot P(T_2 | T_3, \ldots, E_m) \cdots P(E_m)$$

- This expression is sufficiently large if the conditional probabilities in question are large (deductive relations are best, but scarce).
- This is the case if the various theories, models and pieces of evidence in the network support each other well.

Open problem

Is there a single measure that is a good indicator for the joint probability $P(T_1, \ldots, T_n, E_1, \ldots, E_n)$?

I will argue that

- the **coherence** of the set $\{T_1, \ldots, T_n, E_1, \ldots, E_n\}$ is such an indicator, and
- coherence and unification are intimately related.

A formal analogy

- In the *formal epistemology* literature, Shogenji proposed a measure for the coherence of set $S := \{\mathrm{E}_1, \dots, \mathrm{E}_n\}$:

  $$c_S(S) := \frac{P(E_1, \dots, E_n)}{P(\mathrm{E}_1) \times \cdots \times P(\mathrm{E}_n)}$$

  $S := \{\mathrm{E}_1, \dots, \mathrm{E}_n\}$ is more coherent than $S' := \{\mathrm{E}'_1, \dots, \mathrm{E}'_m\}$, if $c_S(S) > c_S(S')$.
- In *philosophy of science*, Myrvold's work entails that a hypothesis H unifies a set of phenomena S $:= \{E_1, \dots, E_n\}$ better than a hypothesis H', if $u_M(S; H) > u_M(S; H')$ with

  $$u_M(S; H) := \frac{P(E_1, \dots, E_n | H)}{P(\mathrm{E}_1 | \mathrm{H}) \times \cdots \times P(\mathrm{E}_n | \mathrm{H})}$$
- Note the formal analogy between the two accounts – both are based on a probabilistic relevance measure.

Problems for Myrvold and Shogenji

Fitelson (2003) presents the following two criticisms of Shogenji's measure:

1. If the E_i are logically equivalent (hence $P(E_i) = p$), then $c_S(S) = p/p^n = p^{1-n}$. This is *unintuitive* as we would expect the coherence of logically equivalent propositions to be maximal and independent of the prior.
2. Shogenji's measure is based on the n-wise independence of the set. It is possible, for example, that two sets differ on all $(n-1)$-wise independencies, but have the same degree of n-wise independence and hence assign the same the (Shogenji) degree of coherence. This is *unintuitive*.

Implications for Myrvold?

Parlance: A hypothesis H *Myrvold-unifies* a set of phenomena S if

$$u_M(S; H) > c_S(S)$$

1. How well does a hypothesis H unify a set of n logically equivalent phenomena with likelihoods $P(E_i|H) = p$? Answer: $u_M(S; H) = p^{1-n}$. This is *unintuitive*.
2. Let's assume that H Myrvold-unifies two sets S and S' with the same degree. H also Myrvold-unifies subsets of S of cardinality $n - 1$, while H does not Myrvold-unify subsets of S' of cardinality $n - 1$. However, Myrvold claims that H unifies S and S' to the same degree. This is *unintuitive*.

Upshot

- We need a unification measure that deals with both problems, i.e. a measure that
  - (i) assigns a maximal (constant) degree of coherence/unification for a set of logically equivalent propositions and
  - (ii) is sensitive to the (in)dependencies implicit in all subsets of S.
- So: Have a look at other coherence measures.
- There is a fairly large literature on this with contributors like Igor Douven and Wouter Meijs, Luca Moretti, Erik Olsson, and others. My favorite account is ...

Bovens and Hartmann's Theory in a Nutshell

- In *Bayesian Epistemology* (OUP 2004), it is argued that a simple explication of the form "coherence is X" (with X = positive relevance, overlap in probability space etc) does not work. We should instead consider the *function* of coherence.
- Coherence plays a *confidence boosting role*, i.e. the more coherent an information set is, ceteris paribus, the greater its posterior probability. The following function generates a partial ordering over a set of information sets:

$$coh_r(S) := \frac{a_0 + (1 - a_0)(1 - r)^n}{\sum_{k=0}^{n} a_k (1 - r)^k}$$

with $a_k := P(n - k \text{ of the } n \text{ propositions in } S \text{ are true})$. E.g. a_0 is the prior probability of S. r is a "dummy" parameter.

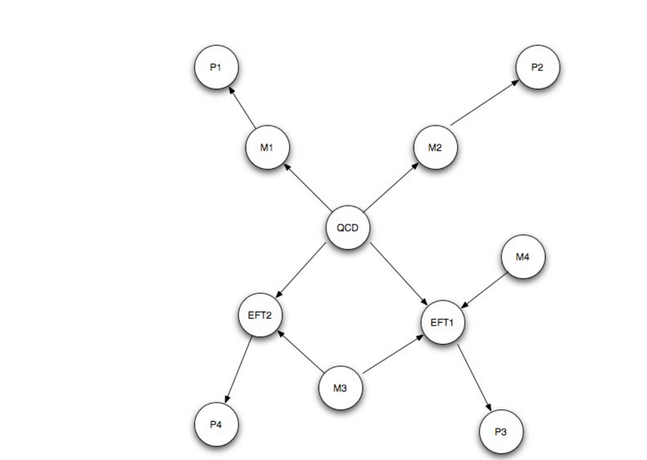
Bovens and Hartmann's Theory (cont'd)

- An information set S is more coherent than an information set S' if $coh_r(S) > coh_r(S')\ \forall r \in (0,1)$.
- Note that $coh_r(S) = 1$ (i.e. the maximally possible value), if all elements of S are logically equivalent. Moreover, it is sensitive to the (in)dependencies implicit in all subsets of S. So our theory deals well with Fitelson's requests.
- With this, various epistemological questions, e.g. under which conditions coherence is truth-conducive, can be addressed. This requires additional modeling assumptions about the information gathering process (but this should not surprise us).

Matching the Descriptive and the Formal

- If we interpret the notion of interrelatedness (as suggested by our case studies) as coherence (in the epistemological sense, then we can apply the formal machinery developed above and use it for explanatory and justificatory purposes.
- One can show, for example, that the more coherent network has the greater joint probability (given certain modeling assumptions).
- One can also study in detail what happens if a positively relevant item is added to the network.
- Using the fibring theory of Gabbay and Williamson, one can also show how accounts at different scales (science as a whole, disciplines, theories etc.) match with each other.

The Structure of Hadron Physics: Interpreted as a Bayesian Network



Conclusions

- Science can be analyzed as a coherent networks of respective units of analysis (disciplines, theories etc.).
- A normative justification for this structure can be given in a Bayesian framework.

Open questions

- Identify specific types of coherent networks and analyze them.
- Study theory change and analyze whether the coherentist picture also applies here (and not just in periods of normal science).
- Which interpretation of probability should be adopted?
- ...

Bayesianism is a progressive research programme with many open questions and interesting problems to address.