

# INDUCTION BEFORE HUME

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The word ‘Induction’ and its cognates in other languages, of which for present purposes the most important is Latin ‘inductio’, have a complex semantic history, as does the Greek ἐπαγωγὴ from which they were derived. Though some of these uses — electromagnetic induction, or the induction of a clergyman into a new benefice — are manifestly irrelevant, others that still diverge significantly from any of the uses current among present-day philosophers and logicians are not. As will soon become apparent, any attempt to write a history that focused solely on the direct ancestors of modern usage would be arduous if not impossible to execute, and deeply unsatisfactory if it could be brought to a conclusion. The net must, at least initially, be cast more widely.

Another potential problem is that there may have been philosophers who discussed problems of inductive inference without using the word ‘induction’ (or its equivalents) at all. The most conspicuous suspect here is David Hume, who has been widely seen — in the twentieth century at least<sup>1</sup> — as an inductive sceptic, even though it is notorious that he rarely used the word, and never in the passages where his inductive scepticism has been located. Whether or not this interpretation of Hume is correct lies outside the scope of this chapter, but it is at least entirely clear that the issue cannot be decided simply from an analysis of Hume’s vocabulary. In the Hellenistic era discussions of non-deductive inference were centred on what became known as inference from signs (*semeiosis*). This was concerned with arguments from the apparent to the non-apparent — either the temporarily and provisionally non-apparent (for example something at a distance), or to the permanently and intrinsically non-apparent (for example invisible bodies such as atoms). How useful it is for modern historians to employ the terminology of induction when dealing with this material is disputed: some do so quite freely, e.g. [Asmis, 1984], while others reject it altogether [Barnes, 1988]. In the present study no attempt will be made to discuss this material in any detail; for some modern accounts see [Burnyeat, 1982; Sedley, 1982; Allen, 2001].

## 1 THE ANCIENT WORLD

Human beings have been making generalisations since time immemorial, and certainly long before any logicians arrived on the scene to analyse what they were doing. Techniques could sometimes go well beyond induction by simple enumeration, as the following remarkable passage from the Old Testament shows:

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<sup>1</sup>[Stove, 1973; Winkler, 1999; Howson, 2000; Okasha, 2001].

And Gideon said unto God, If thou wilt save Israel by mine hand, as thou hast said, Behold I will put a fleece of wool in the floor; and if the dew be on the fleece only, and it be dry upon all the earth beside, then shall I know that thou wilt save Israel by mine hand, as thou hast said. And it was so: for he rose up early on the morrow, and thrust the fleece together, and wringed the dew out of the fleece, a bowl full of water. And Gideon said unto God, Let not thine anger be hot against me, and I will speak but this once: let me prove, I pray thee, but this once with the fleece; let it now be dry only upon the fleece, and upon all the ground let there be dew. And God did so that night: for it was dry upon the fleece only, and there was dew on all the ground. (Judges, vi. 36–40).

Neither the writer of this passage nor his readers had ever read Mill, or heard of the Method of Agreement or Method of Difference, but few could have found Gideon's procedures difficult to comprehend. As Locke was to comment sardonically, 'God has not been so sparing to Men to make them barely two-legged Creatures, and left it to *Aristotle* to make them Rational' (*Essay*, IV. xvii. 4; [Locke, 1975, p. 671]).

It was, nevertheless, Aristotle who was the first philosopher to give inductive reasoning a name and to provide an account, albeit a brief and imperfect one, of what it was and how it worked. The name chosen was ἐπαγωγή (*epagoge*), derived from the verb ἐπάγειν, variously translated, according to context, as to bring or lead in, or on. Like 'induction' in modern English, *epagoge* had (and continued to have) a variety of other, irrelevant meanings: Plato had used it for an incantation (*Republic* 364C), and Aristotle himself employed it for the ingestion of food (*De Respiratione* 483a9).

### 1.1 Socrates and Plato

Although none of Aristotle's predecessors had anticipated him in using the term *epagoge* for inductive arguments, he had himself picked out Socrates for his use of what Aristotle called ἐπαγωγικούς λόγους (*Metaphysics* 1078b28). Though Aristotle would have had testimony about Socrates' activities that has since been lost, there can be little doubt that his main source of information was Plato. In the early dialogues, Socrates was often portrayed as using modes of argument that Aristotle would certainly have classed as *epagoge*, for example in *Protagoras* 332C, where Socrates is reporting his interrogation of Protagoras:

Once more, I said, is there anything beautiful?  
 Yes.  
 To which the only opposite is the ugly?  
 There is no other.  
 And is there anything good?  
 There is.

To which the only opposite is the evil?  
 There is no other.  
 And there is the acute in sound?  
 True.  
 To which the only opposite is the grave?  
 There is no other, he said, but that.  
 Then every opposite has one opposite only and no more?  
 He [Protagoras] assented. [Plato, 1953, vol. I, p. 158]

Here and elsewhere (e.g. *Charmides* 159–160; *Ion* 540) the conclusion is a philosophical one that could have been grasped directly by someone intelligent and clear-sighted enough. Plato was concerned with truths such as these, not with empirical generalisations involving white swans or other sensory particulars [Robinson, 1953, pp. 33–48; McPherran, 2007].

## 1.2 Aristotle

Aristotle's theory of induction — or to put it more neutrally, of *epagoge*, since there is disagreement even about the most appropriate translation of that term — has long been a matter of controversy. It is widely regarded as incomplete and in various respects imperfect: one modern commentator has referred to 'the common belief [that] Aristotle's concept of induction is incomplete, ill-conceived, unsystematic and generally unsatisfactory', at least in comparison with his theory of deduction [Upton, 1981, p. 172]. Though not everyone might agree with this, it is clear that there is no consensus either about what exactly Aristotle was trying to do, or about how successful he was.<sup>2</sup>

When Aristotle used the word *epagoge* to characterise his own arguments, his employment of the term is thoroughly Socratic, or at least Platonic; the arguments were seldom empirical generalisations, or anything like them. The following passage from *Metaphysics* I is in this respect entirely typical:

That contrariety is the greatest difference is made clear by induction [ἐκ τῆς ἐπαγωγῆς]. For things which differ in genus have no way to one another, but are too far distant and are not comparable; and for things that differ in species the extremes from which generation takes place are the contraries, and the distance between extremes — and therefore that between the contraries — is greatest. (1055a5–10).

Similar remarks can be found elsewhere in the same book, e.g. in 1055b17 and 1058b9.

Aristotle discussed *epagoge* in three passages, none of them very long. The earliest is in *Topics* A12, where dialectical arguments are divided into two kinds,

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<sup>2</sup>A selection of diverse views can be found in [Kosman, 1973; Hamlyn, 1976; Engberg-Pedersen, 1979; Upton, 1981; Caujolle-Zaslavsky, 1990; McKirahan, 1992, pp. 250–7; De Rijk, 2002, pp. 140–8].

*sylogismos* and *epagoge*. The meaning of the former term is certainly broader than ‘syllogism’ as now generally understood, and as the word is used in Aristotle’s later writings; it can probably best be translated as ‘deduction’. *Epagoge* is characterised quite briefly:

Induction is the progress from particulars to universals; for example, ‘If the skilled pilot is the best pilot, and the skilled charioteer the best charioteer, then in general the skilled man is the best man in any particular sphere.’ Induction is more convincing and more clear and more easily grasped by sense perception and is shared by the majority of people, but reasoning [*sylogismos*] is more cogent and more efficacious against argumentative opponents (105a12–19).

The first part of this subsequently became the standard definition of induction in the Middle Ages and Renaissance. It is natural for a modern reader to interpret it as meaning that induction is the mode of inference that proceeds from particular to universal propositions, but the Greek does not quite say this. Induction is merely the passage (ἔφοδος) from individuals to universals, τὰ καθόλου, and in other places (notably *Posterior Analytics* B19) these universals would seem to be, or at least to include, universal concepts. It should also not be automatically assumed that ‘ἔφοδος’ means inference in any technical sense [De Rijk, 2002, pp. 141–4].

Aristotle’s longest account of *epagoge* is in *Prior Analytics* B23:

Now induction, or rather the syllogism which springs out of induction [ὁ ἐξ ἐπαγωγῆς συλλογισμὸς], consists in establishing syllogistically a relation between one extreme and the middle by means of the other extreme, e.g. if *B* is the middle term between *A* and *C*, it consists in proving through *C* that *A* belongs to *B*. For this is the manner in which we make inductions. For example let *A* stand for long-lived, *B* for bileless, and *C* for the particular long-lived animals, e.g. man, horse, mule. *A* then belongs to the whole of *C*: *for whatever is bileless is long-lived*.<sup>3</sup> But *B* also (‘not possessing bile’) belongs to all *C*. If then *C* is convertible with *B*, and the middle term is not wider in extension, it is necessary that *A* should belong to *B*. For it has already been proved that if two things belong to the same thing, and the extreme is convertible with one of them, then the other predicate will belong to the predicate that is converted. But we must apprehend *C* as made up of all the particulars. For induction proceeds through an enumeration of all the cases. (68b15–29).

This is not an easy passage to understand, and has been the subject of much discussion. Aristotle appears to be applying his method of conversion, devised as

<sup>3</sup>The phrase given here in italics makes no sense here; it may be an interpolation and if so should be excised [Aristotle, 1973, p. 514], even though there is no manuscript support for doing this [Ross, 1949, p. 486].

part of his account of syllogisms, to a case where it is not obviously applicable: hence the mention of middle terms. The crucial step in the argument is that  $B$  belongs to all  $C$ , i.e. that every long-lived animal is bileless. This could mean that every individual long-lived animal is bileless, or it could mean that every species of such animals is bileless. The latter seems to be indicated by the examples given — man, horse, mule, rather than (say) Socrates, Bucephelas etc. If so, then Aristotle appears to have been giving an example of what has subsequently come to be termed perfect (i.e. complete) induction: an inference from a finite sample that is sufficiently small for all the particular cases to be examined. This might seem to be what is indicated by the final remark, that ‘induction proceeds through an enumeration of all the cases’, but here (as often) the Oxford translation supplies words not present in the Greek, which merely says ‘for induction [is] through all’, ἡ γὰρ ἐπαγωγὴ διὰ πάντων.

It is perhaps significant here that the proposition being proved — that all bileless animals are long-lived — is a generalisation about the natural world, and therefore very unlike the propositions argued for by Socrates in the early Platonic dialogues. It is manifestly not something that could in principle be grasped immediately by intuition. The same is true of another proposition described as having been derived by induction: in *Posterior Analytics* A13 (78a30–b4) Aristotle gave a celebrated example of a scientific demonstration:

- (1) The planets do not twinkle.  
 (2) Whatever does not twinkle is near.  
 Therefore (3) The planets are near.

This counts as a demonstration, as distinct from a merely valid syllogism, because it states the cause: it is *because* the planets are near (i.e. nearer than the fixed stars) that they do not twinkle. Premise (2) is described as having been reached ‘by induction or through sense-perception’ (78a34–5), though the same must in fact be true also of premise (1). For (1) the argument is straightforward and unproblematic — Mercury does not twinkle, Venus does not twinkle, etc. — but for (2) it is not. There is clearly no difficulty in assembling a long list of particular non-twinkling objects that are also nearby, but how could the general proposition that *all* such objects are nearby be established? If it is supposed to be the conclusion of an inductive argument, then the enumeration is manifestly incomplete, and the inference correspondingly fallible.

The demonstrations analysed in the *Posterior Analytics* are syllogistic arguments (here ‘syllogism’ is being used in the strict sense) which proceed from premises that are ‘true, primary, immediate, better known than, prior to, and causative of their conclusion’ (71b20–2). All these premises are universal in form, and this raises an obvious question: if the primary premises from which demonstrations proceed cannot themselves be demonstrated, how are they to be known? It was an issue that Aristotle deferred until the final chapter of the second book. The problem is stated quite clearly at the beginning of the chapter, but the discussion that follows at first sight seems rather puzzling: rather than discussing

inductive arguments, Aristotle appears to be trying to account for the acquisition of universal concepts — from the perception of several individual men to the species *man*, and then to the genus *animal* (100a3–b3). He then commented (this is the only place in which the word *epagoge* occurs in the whole chapter): ‘Thus it is clear that it is necessary for us to come to know the first principles by induction, because this is also the way in which universals are put into us by sense perception’ (100b3–5).

The whole passage is undeniably difficult, and has been diversely interpreted, as the two main English commentaries on the *Posterior Analytics* show. Sir David Ross took it that Aristotle was concerned with both concept formation and induction, and treated them together because ‘the formation of general concepts and the grasping of universal propositions are inseparably interwoven’ [Ross, 1949, p. 675]. Jonathan Barnes, on the other hand, held that ‘Here “induction” is used in a weak sense, to refer to any cognitive progress from the less to the more general . . . Thus construed, 100b3–5 says no more than that concept acquisition proceeds from the less to the more general.’ [Barnes, 1975, p. 256]. On Barnes’s reading, the passage is not concerned with the inference from singular to universal propositions at all.

This is not a dispute that can easily be resolved: the relevant texts are quite short, and all the participants in the debate are thoroughly familiar with them. My own inclination is to side with Ross. Aristotle’s position here is very different from that found in a later empiricist like Locke. Locke had an account of how humans — unlike the other animals that he called ‘brutes’ — had a capacity to frame abstract general ideas from the ideas of particular things given in perception [Locke, 1975, pp. 159–60], but this process had nothing to do with an inductive ascent from particular to universal propositions, about which Locke said virtually nothing. For Aristotle what comes to rest in the soul (more specifically, in the intellect) is not a mere Lockean abstract general idea, a particular entity that has the capacity to function as a universal sign, but rather a real universal thing, a form freed from matter and thereby de-individuated. This is why the same psychological process can be used to explain both the acquisition of universal concepts and the knowledge of first principles. In the *Posterior Analytics* the account of this is little more than a sketch, but it was subsequently fully worked out by Aristotle’s followers in late antiquity and in the Middle Ages.

There is no hint whatever in Aristotle that *epagoge* is merely one of several ways by which we can gain knowledge of first principles. The view found in many modern empiricists that while some universal truths are known — or at least receive some degree of evidential support — a posteriori, by induction, others (for example Euclid’s axiom that all right angles are equal) are known a priori, is entirely foreign to his way of thinking. For Aristotle it is impossible to view ( $\theta\epsilon\omega\rho\eta\sigma\alpha\iota$ ) universals except through induction (*Posterior Analytics* 81b2).

In all the passages mentioned so far, *epagoge* is treated as a process leading to universals, whether concepts, or propositions, or both. This is explicit in the definition in the *Topics*, but it can also be seen in the *Prior* and the *Posterior Analytics*. Often, however, and especially in the practical affairs of life, we are

concerned with reasoning from particulars to other particulars — whether the sun will rise tomorrow, whether this loaf of bread will nourish me, and so on. Aristotle was, of course, well aware that we do this, and classified such inferences as ‘examples’ (*paradeigmata*). What is less clear is whether *paradeigma* is a type of induction, or whether it is a different kind of argument, resembling induction in various ways, but not a sub-variety of it.

In *Prior Analytics* B24, the chapter immediately after the chapter on induction, there is an account of *paradeigmata*. To give one specimen of such an argument, Athens against Thebes and Thebes against Phocis are both cases of wars against neighbours; the war against Phocis was bad for Thebes, so a war against Thebes would be bad for Athens (68b41–69a13). The inference might appear to proceed via a more general principal that war against neighbours is always bad (69a4, 6), which would make it an application of induction: a two-part argument involving an inductive ascent to a generalisation followed by a deductive descent to a particular case. Aristotle, however, insisted that the two kinds of inference were distinct: example is not reasoning from part to whole or from whole to part, but from part to part (69a14–15). Induction proceeds by an examination of all the individual cases (ἐξ ἀπάντων τῶν ἀτόμων), while example does not (69a16–19).

In Aristotle’s *Rhetoric*, however, induction and example seem much closer, if not identical:

just as in dialectic there is induction on the one hand and syllogism or apparent syllogism on the other, so it is in rhetoric. The example is an induction, the enthymeme<sup>4</sup> is a syllogism, and the apparent enthymeme is an apparent syllogism. I call the enthymeme a rhetorical syllogism and the example a rhetorical induction. Every one who effects persuasion through proof does in fact use either enthymemes or examples: there is no other way. And since every one who proves anything at all is bound to use either syllogisms or inductions (and this is clear to us from the *Analytiks*), it must follow that enthymemes are syllogisms and examples are inductions (1356b1–10).

The exhaustive division of all arguments into either *sylogismos* or *epagoge* is not peculiar to the *Rhetoric*: it can be found in both parts of the *Analytiks* (68b13–14, 71a5–6), as can the identification of enthymeme and example as their rhetorical counterparts (71a9–11). One very plausible way of interpreting this is that enthymeme and example are not sub-varieties of *sylogismos* and *epagoge*, still less entirely different types of argument, but rather instances of *sylogismos* and *epagoge* ‘when these occur in a rhetorical speech rather than in a dialectical argument’ [Burnyeat, 1994, p. 16]. If this is done, however, the notion of *epagoge* must be broadened to include most if not all non-deductive argument, since one thing that is absolutely certain about *paradeigma* is that it concerns arguments from particulars to particulars.

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<sup>4</sup>Aristotle’s account of enthymeme is complex and has often been misunderstood, but lies outside the scope of this chapter; for a penetrating modern analysis, see [Burnyeat, 1994].

None of Aristotle's surviving works contains a detailed and systematic account of induction, and there is no evidence that one was ever produced. Why this should have been the case is not obvious, given the potential importance of such reasoning in his theory of knowledge, but one explanation may be that the separation of form and content, which had been central to his analysis of the syllogism, was (and still remains) more difficult to achieve in the case of induction. At all events, Aristotle did not bequeath to his successors an account of induction that was in any way comparable to his treatment of the syllogism.

### 1.3 *Hellenistic and later Greek accounts*

In the three centuries that followed Aristotle's death, his technical writings were not much studied outside the (declining) Peripatetic school, and the terms that he had devised were replaced by others. The problems involved in inference from particular to universal propositions were raised occasionally, but they seem not to have become the central issue of discussion, unlike the problems of inference from signs.

#### *Alcinous*

The lack of any serious interest in induction among the Platonists is indicated by the extremely brief treatment in one of the few philosophical textbooks to survive, the *Handbook of Platonism* (*Didaskalikos*) attributed to a certain Alcinous, often identified with the Middle Platonist Albinus (2nd century AD):

Induction is any logical procedure which passes from like to like, or from the particular to the general. Induction is particularly useful for activating the natural concepts (*Didaskalikos*, 6.7; [Dillon, 1993, p. 10]).

The last remark may allude to the well-known passage in the *Meno* where the slave boy is being led to reveal his innate knowledge of geometry [Dillon, 1993, p. 77]. One finds here a characteristic blend of Platonism and Aristotelianism: the role of induction is to provide particular examples that can bring to full consciousness the concepts implanted in us by nature.

#### *Diogenes Laertius*

Two other Greek writers from the Roman period had rather more to say about induction: the biographer Diogenes Laertius (early 3rd century?), and the Pyrrhonian sceptic, Sextus Empiricus (late 2nd or early 3rd century?). Neither was an original thinker, and indeed Diogenes was barely a thinker at all, but rather a scissors-and-paste compiler whose labours would have been ignored by posterity had they not resulted in the only extensive compendium of philosophical biographies to have survived from antiquity.

Diogenes' remarks on induction are in his life of Plato (III. 53–55). *Epagoge* is defined as an argument in which we infer from some true premises a conclusion resembling them. There are two varieties: from opposites (*κατ' ἐναντίωσιν*), and from implication (*ἐκ τῆς ἀκολουθίας*). The former is a mode of argument that bears little resemblance to any modern notion of induction:

If man is not an animal he will be either a stick or a stone. But he is not a stick or a stone, for he is animate and self-moved. Therefore he is an animal. But if he is an animal, and if a dog or an ox is also an animal, then man by being an animal will be a dog and an ox as well.

The first part of this is clear enough — it seems that either Diogenes or his source was using an ancient version of the question ‘Animal, Vegetable or Mineral?’ — but the last part is considerably more obscure. The second kind of induction is much more familiar. There are two sub-varieties: one, described as belonging to rhetoric, in which the argument is from particulars to other particulars, and the other, belonging to dialectic, in which it is from particulars to universals. The former is clearly the Aristotelian *paradeigma*, though that term was not used. An instance of the latter is the argument that the soul is immortal:

And this is proved in the dialogue on the soul [presumably the *Phaedo*] by means of a certain general proposition, that opposites proceed from opposites. And the general proposition is established by means of some propositions which are particular, as that sleep comes from waking and vice-versa, and the greater from the less and vice-versa.

These are not examples of empirical generalisations.

### *Sextus Empiricus*

Among the immense range of sceptical arguments preserved and deployed by Sextus Empiricus, inductive scepticism is inconspicuous, though not wholly absent. In the *Outlines of Pyrrhonism* II. 204 inductive arguments were dismissed in a very cursory, almost contemptuous, manner:

It is also easy, I consider, to set aside the method of induction [*τὸν περὶ ἐπαγωγῆς τρόπον*]. For, when they propose to establish the universal from the particulars by means of induction, they will effect this by a review either of all or of some of the particular instances. But if they review some, the induction will be insecure, since some of the particulars omitted in the induction may contravene the universal; while if they are to review all, they will be toiling at the impossible, since the particulars are infinite and indefinite. Thus on both grounds, as I think, the consequence is that induction is invalidated.<sup>5</sup> [Sextus, 1967, p. 283].

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<sup>5</sup>Literally, ‘shaken’, or ‘made to totter’.

Another passage a few pages earlier (II. 195) supplies a little more detail:

Well then, the premiss ‘Every man is an animal’ is established by induction from particular instances; for from the fact that Socrates, who is a man, is also an animal, and Plato likewise, and Dion and each one of the particular instances, they think it is possible to assert that every man is an animal. . . [Sextus, 1967, p. 277].

Sextus was not persuaded: if even a single counter-example can be found, the universal conclusion is not sound (ὕγιής, i.e. healthy), ‘thus, for example, when most animals move the lower jaw, and only the crocodile the upper, the premiss “Every animal moves the lower jaw” is not true.’ [Sextus, 1967, p. 277]. At first sight this differs from the familiar modern textbook example of ‘All swans are white’ being falsified by the observation of a single individual black swan, but in fact the differences are small. In the case of the swans, what makes the falsification effective is that it was a *species* of black swans that was discovered. Logically speaking, a single negative instance can falsify a universal proposition; in practice it usually would not, as a variety of what Imre Lakatos called ‘monster-barring’ stratagems would come into play.

It is very unlikely that the generalisation about how animals move their jaws, with the crocodile as an exception, was original to Sextus: the same example can be found in Apuleius’ *Peri Hermeneias* [Apuleius, 1987, p. 95]. It had probably long been a stock example, repeated from author to author.

#### *Alexander of Aphrodisias*

The view that conclusions drawn from inductive arguments are not conclusively established was not peculiar to the sceptics — indeed it can be found among the Aristotelians themselves, notably the late second-century commentator Alexander of Aphrodisias. On the passage in *Topics* 105a10ff quoted above, Alexander observed:

So induction has the quality of persuasiveness; but it does not have that of necessity. For the universal does not follow by necessity from the particulars once these have been conceded, because we cannot get something through induction by going over all the particular cases, since the particular cases are impossible to go through [Alexander, 2001, p. 93].

As this and other remarks to be quoted in what follows show quite clearly, it is utterly mistaken to suppose that Hume was the first person to notice that inductive arguments are not deductively valid, and that any universal generalisation which covers a field that is either infinite or too large to survey completely is vulnerable to counter-examples. To suppose this would be unfair both to Hume, who was certainly doing something more radical and much less banal, and to his predecessors, who had taken the fallibility of such inferences for granted.

## 1.4 Roman philosophy

### *Cicero and the rhetorical tradition*

The Romans, unlike their medieval successors, had little interest in logic as a technical discipline,<sup>6</sup> but rhetoric was a central — perhaps *the* central — element of their educational curriculum.

When philosophy began to be written in Latin, a new technical vocabulary needed to be devised. Who introduced the term ‘inductio’ for *epagoge* is not now known, but in the surviving corpus of Latin literature the word first appears with this sense in a youthful work by Cicero, *De Inventione*. Here it is described as a form of argument in which the speaker first gets his opponent to agree on some undisputed propositions, and then leads him to assent to others resembling them. In the example Cicero gave, Pericles’ sharp-witted mistress Aspasia is interrogating the wife of a certain Xenophon (not the historian):

‘Please tell me, if your neighbour had a better gold ornament than you have, would you prefer that one, or your own?’ ‘That one’, she said. ‘And if she had clothes or other finery more expensive than you have, would you prefer yours or hers?’ ‘Hers, of course’, she replied. ‘Well then, if she had a better husband than you have, would you prefer yours or hers?’ At this, the woman blushed. (I. 55).

Clearly this is not a specimen of inductive generalisation, but rather of what Aristotle called *paradeigma*. Cicero had little interest in the kinds of generalisation that might be made by a natural philosopher: his concern, here as elsewhere, was with the strategies that can be used in public speaking or in a court of law. In a later rhetorical treatise, the *Topics*, induction is mentioned very briefly as merely one variety of a more extensive class of arguments from similarity. The example Cicero gave — that if honesty is required of a guardian, a partner, a bailee and a trustee, it is required of an agent (*Topics*, 42) — is described as an *epagoge* (the Greek term was used), but it is clearly a case of what Aristotle had called *paradeigma*. In the rhetorical tradition, it was the analysis and employment of arguments of this type that attracted most interest.

Cicero’s account of induction was followed by the writers of rhetorical treatises and textbooks, notably Quintilian’s *Institutio Oratoria*, V. x. 73, xi. 2 [Quintilian, 1921, vol. II, pp. 241, 273], though the treatment is fairly cursory: induction was merely one rather unimportant variety of reasoning, less deserving of extended analysis than either arguments from signs or examples. This subsumption of induction into the theory of rhetoric had the unwelcome result (for analytically minded historians of philosophy) that what they have thought of as the Problem of Induction — the enquiry into how (if at all) universal propositions can be proved, or

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<sup>6</sup>Though the aversion was by no means universal: see [Barnes, 1997, ch. 1].

at least made probable,<sup>7</sup> from evidence of particular cases — was never properly raised, let alone answered.

### *Boethius*

It was only in the final twilight of the ancient world, after the fall of the Empire in the west, that Aristotle's writings started to be translated into Latin. Boethius had planned to translate and comment on the entire corpus, but by the time of his premature death only a small part of this exceedingly ambitious project had been completed. The only translations that have survived were of the *Categories* and *De Interpretatione*, but Boethius' own logical writings gave his early medieval successors some information about the content of Aristotle's other works on logic. Induction was dealt with fairly briefly in *De Topicis Differentiis* [Stump, 1978, pp. 44–46], being described in Aristotelian rather than Ciceronian terms as a progression from particulars to universals. This is taken directly from Aristotle's account in the *Topics*, as was the example given to illustrate it: just as a pilot should be chosen on the basis of possessing the appropriate skill rather than by lot, and similarly with a charioteer, so generally if one wants something governed properly one should choose someone on the basis of their skill. The main historical importance of Boethius' account is not that it added anything to earlier analyses — it did not — but that it provided his early medieval readers with information then unavailable from any other source.

### *Summary*

It is striking that no sustained discussion of inductive reasoning has survived from the ancient world. Of course the vast majority of Greek and Roman philosophical works have perished, and are accessible only from fragments quoted by other writers, or often not at all. If more had been preserved, then the patchy and episodic account given above could unquestionably have been made considerably longer and more detailed. There is nevertheless no sign that a major and systematic account of inductive reasoning has been lost: among the many lists of works given by Diogenes Laertius there is no sign of any treatise with the title *Peri Epagoges* or something similar.

It would appear, therefore, that induction was not something that any of the ancients regarded as one of the central problems of philosophy. Several reasons for this state of affairs can be discerned.

One is that the general drift of philosophy, especially in late antiquity, was also away from the kind of systematic empirical enquiry practised by Aristotle and his immediate successors. Plotinus, for example, used the word *epagoge* only twice, once for an argument to show that there is nothing contrary to substance, and once for an argument that whatever is destroyed is composite (*Enneads*, I. 8. 6;

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<sup>7</sup>On the meaning of *probabilis* and related terms in Cicero and other ancient authors, see [Glucker, 1995]. On subsequent history, see [Hacking, 1975; Cohen, 1980; Franklin, 2001].

II. 4. 6). The kind of understanding gained through empirical generalisation was too meagre and unimportant for the modes of argument leading to it to merit sustained analysis.

Another reason is that interest in the systematic investigation of the natural world was intermittent and localised. It did not help that the scientific discipline in which the greatest advances were made had been mathematical astronomy, and this was not a field where the problems posed by inductive reasoning would have surfaced, still less become pressing. Constructing a model for the motions of a planet was a highly complex business, but it did not involve generalisation from data in the form 'this  $A$  is  $B$ ' and 'this  $A$  is  $B$ ' to 'every  $A$  is  $B$ '. Ptolemy, indeed, seems to have felt so little urge to generalise that his models for the individual planets are all given separately, and (in the *Almagest* at least) not integrated into a single coherent system

Finally, the centrality of rhetoric in ancient education meant that when inductive arguments were discussed, they tended to be evaluated for their persuasiveness, not for their logical merits. Inductive arguments became almost lost in a mass of miscellaneous un-formalised arguments that were not investigated for their validity, or any inductive analogue thereof, but for their plausibility in the context of a speech.

## 2 THE MIDDLE AGES

2.1 *Arabic accounts*

Two civilisations inherited the legacy of ancient philosophy. Starting in the late eighth century, a large part of the philosophical literature that had been fashionable in late antiquity was translated into Arabic, including most of the corpus of Aristotle's writings and many of the works of his commentators. The accounts of induction in the *Prior Analytics*, the *Posterior Analytics* and the *Topics* became the starting point of subsequent treatments. Very little of the Greek technical terminology was directly transliterated, a notable exception being the word for philosophy itself (*falsafah*). *Epagoge* was translated as *istiqrâ*, a word whose root meaning was investigation or examination.<sup>8</sup>

*Al-Fârâbî*

The first Arabic writer to give a systematic account of induction was al-Fârâbî (c.870–c.950) [Lameer, 1994, pp. 143–154, 169–175]. His conception of induction differed in one important respect from Aristotle's. According to Joep Lameer:

For Aristotle, induction is the advance from a number of related particular cases to the corresponding universal. In opposition to this, al-Fârâbî explains induction in terms of an examination of the particulars. This view must be taken to be a natural consequence of the fact that in the Arabic *Prior Analytics*, *epagôgê* was rendered as *istiqrâ* ('collection' in the sense of a scrutiny of the particulars). [Lameer, 1994, p. 173, cf. p. 144].

This conception of induction as proceeding by a one-by-one examination of the particulars had the consequence that inductions have full probative force only when they are complete [Lameer, 1994, pp. 144, 147].

Al-Fârâbî also made a distinction between induction and what he called methodic experience (*tajriba*, equivalent to Greek *empeiria*):

methodic experience means that we examine the particular instances of universal premises to determine whether a given universal is predicable of each one of the particular instances, and we follow this up with all or most of them until we obtain necessary certainty, in which case that predication applies to the whole of that species. Methodic experience resembles induction, except the difference between methodic experience and induction is that induction does not produce necessary certainty by means of universal predication, whereas methodic experience does. [McGinnis and Reisman, 2007, p. 67].

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<sup>8</sup>For insight into the meaning of Arabic terminology I am grateful to my colleague Peter Adamson.

Induction is inferior to methodic experience because it does not uncover necessary truths or lead to certain knowledge.

*Avicenna*

The same distinction between induction and methodic experience appears in Avicenna (Ibn Sina, 980–1037) [McGinnis, 2003], [McGinnis, 2008]. In his main philosophical work, *The Cure* (Book of Demonstration, I. 9. §§ 12, 21), induction was described as inferior to methodic experience, in that unless it proceeds from an examination of all the relevant cases, it leads only to probable belief; [McGinnis and Reisman, 2007, pp. 149, 152]).

Methodic experience is not like induction . . . methodic experience is like our judging that the scammony plant [*Convolvulus scammonia*] is a purgative for bile; for since this is repeated many times, it stops being a case of something that occurs by chance, and the mind then judges and grants that it is characteristic of scammony to purge bile. Purging bile is a concomitant accident of scammony. [McGinnis and Reisman, 2007, p. 149].

The Aristotelian background is apparent here: events due merely to chance do not recur regularly, and a regular succession is therefore a sign that something is occurring naturally:

Now one might ask: ‘This is not something whose cause is known, so how are we certain that scammony cannot be sound of nature, and yet not purge bile?’ I say: Since it is verified that purging bile so happens to belong to scammony, and that becomes evident by way of much repetition, one knows that it is not by chance, for chance is not always or for the most part. Then one knows that this is something scammony necessarily brings about by nature, since there is no way it can be an act of choice on the part of scammony. [McGinnis and Reisman, 2007, p. 149]

To use the language of more recent philosophers, we know a priori that the physical world is full of natural law-like regularities, and we merely need enough experience to show that the apparent regularity we are considering is one of these, and not something due purely to chance. Even if this is granted, however, methodic experience does not produce certainty: there is always the risk of coming up with a generalisation that is too wide:

We also do not preclude that in some country, some temperament and special property is connected with or absent from the scammony such that it does not purge. Nevertheless, the judgment based on methodic experience that we possess must be that the scammony commonplace among and perceived by us purges bile, whether owing to its essence

or a nature in it, unless opposed by some obstacle. [McGinnis and Reisman, 2007, p. 151]

Another problem is effectively identical to the white-swan problem of modern textbooks:

Were we to imagine that there were no people but Sudanese, and that only black people were repeatedly perceived, then would that not necessarily produce a conviction that all people are black? On the one hand, if it does not, then why does one repetition produce such a belief, and another repetition does not? On the other hand, if the one instance of methodic experience does produce the belief that there are only black people, it has in fact produced an error and falsehood. [McGinnis and Reisman, 2007, p. 150]

It was a very pertinent question, and Avicenna's response was rather opaque:

you can easily resolve the puzzle concerning the Sudanese and their procreation of black children. In summary form, when procreation is taken to be procreation by black people, or people of one such country, then methodic experience will be valid. If procreation is taken to be that of any given people, then methodic experience will not end with the aforementioned particular instances; for that methodic experience concerned a black people, but people absolutely speaking are not limited to black people. [McGinnis and Reisman, 2007, p. 150]

Though Avicenna's writings had an immense influence on the philosophers in the universities of medieval Europe, this particular work was never translated into Latin. The purgative powers of scammony, however, became a stock example in scholastic discussions, probably through its use in Avicenna's medical writings, which had an immense influence on medical education in the Latin west [Weinberg, 1965, pp. 134–135].

## 2.2 *The Latin West*

Boethius' categorisation of induction as a progression from particulars to universals was only one definition current during the Middle Ages. Another was a more rhetorical definition, derived ultimately from Cicero and transmitted by authors such as Victorinus and Alcuin, that made no mention of universality. Alcuin defined induction as an argument that from certain things proves uncertain ones, and compels the assent of the unwilling [Halm, 1863, p. 540].<sup>9</sup>

The reception and translation of the main body of Aristotle's writings into Latin during the course of the twelfth and thirteenth centuries, initially from Arabic,

<sup>9</sup>Inductio est oratio quae per certas res quaedam incerta probat et nolentem ducit in assensionem, *Disputatio de Rhetorica et De Virtutibus*, 30. According to Victorinus, Inductio est oratio, quae rebus non dubiis captat adensiones eius, quicum instituta est, *Explanationum in Rhetoricam M. Tullii Ciceronis Libri Duo*, I. 31, [Halm, 1863, p. 240].

but subsequently directly from Greek, focused the attention of philosophers in the universities on the logical rather than the rhetorical tradition. There are two main locations for discussions of induction in the works of the schoolmen. One was in commentaries and questions on the *Prior* and *Posterior Analytics*, the other in general treatises on logic and logic textbooks, though few of these dealt with it at length.<sup>10</sup>

*Robert Grosseteste*

One of the most elaborate and most interesting commentaries on the *Posterior Analytics* was one of the first, written by Robert Grosseteste before 1230 [Hackett, 2004, p. 161]. Grosseteste's account of induction was based closely on the final chapter of the *Posterior Analytics*, though the specific example he used came from Avicenna:

For when the senses several times observe two singular occurrences, of which one is the cause of the other or is related to it in some other way, and they do not see the connections between them, as, for example, when someone frequently notices that the eating of scammony happens to be accompanied by the discharge of red bile and does not see that it is the scammony that attracts and withdraws the red bile, then from constant observation of these two observable things it begins to form [*estimare*] a third, unobservable thing, namely that scammony is the cause that withdraws the red bile [Grosseteste, 1981, pp. 214–215; Crombie, 1953, pp. 73–74]

This looks much more like the advancing of a causal hypothesis than a specimen of inductive generalisation. The next part of Grosseteste's account followed Aristotle closely: repeated perceptions are stored in the memory, and this in turn leads to reasoning:

Reason begins to wonder and consider whether things really are as the sensible recollection says, and these two lead the reason to the experiment [*ad experientiam*], namely, that scammony should be administered after all other causes purging red bile have been isolated and excluded. But when he has administered scammony many times with the sure exclusion of all other things that purge red bile, then there is formed in the reason this universal, namely that all scammony of its nature withdraws red bile; and this is the way in which it comes from sensation to a universal experimental principle. [Grosseteste, 1981, p. 215; Crombie, 1953, p. 74]

If the conclusion had merely been that all scammony draws out red bile, then the argument would be a clear and unproblematic case of inductive generalisation. In

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<sup>10</sup>Little has been written specifically on medieval accounts of induction, but for two short general surveys, see [Weinberg, 1965; Bos, 1993].

fact the conclusion is stronger than this: that scammony of its nature [*secundum se*] draws out red bile. No doubt Grosseteste would have replied that the inference would only be safe if the power of drawing out bile really was part of the nature of scammony.

The framework and most of the details of Grosseteste's account are plainly Aristotelian, but there is one important difference. In the *Posterior Analytics* a plurality of memories constitute a single experience (*empeiria*), and this, unlike the memories from which it had arisen, is universal (100a5–6); there is no suggestion whatever that anything that we would now describe as an experiment needs to be undertaken. Grosseteste's procedure was much more interventionist: scammony is to be administered in a variety of situations in which all the other substances that are known to purge bile have been excluded, and it is this systematic variation of the circumstances that provides the justification for the universal conclusion.

### *William of Ockham*

The most detailed account of induction by any of the writers on logic was given by Ockham in his *Summa Logicae*, Part III, section iii, chapters 31–36. In the first of these, induction was defined in the manner of Aristotle and Boethius, as a progression from singulars to a universal [Ockham, 1974, p. 707]. In both the premises and the conclusion the predicate remains the same, and variation occurs merely in the subject: for example 'This [man] runs, that [man] runs, and so on for other singulars [*et sic de singulis*], therefore every man runs', or 'Socrates runs, Plato runs, and so on for other singulars, therefore every man runs' [Ockham, 1974, p. 708]. In all these examples Ockham was concerned with propositions ascribing a predicate to an individual (Socrates, this man, that white thing), and not a species. This is fully consonant with his thoroughgoing nominalism: only individuals exist, and universals are merely signs that represent them.

In the chapters that follow Ockham gave a series of rules for sound and unsound inductive inferences. He began by considering non-modal propositions about present states of affairs (*de praesenti et de inesse*). There are three rules for these:

1. Every true universal proposition has some true singular.
2. If all the singulars of some universal proposition are true, then the universal is true.
3. If a negative universal proposition is false, then it follows that at least one of its singulars is false.

[Ockham, 1974, pp. 708–709]

The first of these points to a fundamental difference between medieval and modern post-Fregean logic, in which 'Every *A* is *B*' does not imply that 'Some *A* is *B*'. The second rule might seem obvious, but as becomes apparent in the chapters that

follow, there are types of proposition for which Ockham thought that it did not apply.

For some modal propositions — those *in sensu divisionis*<sup>11</sup> — the same rules apply: just as we can draw the conclusion that ‘Every man runs’ from ‘Socrates runs’, ‘Plato runs’, etc., so we can make the inference ‘Socrates is contingently an animal, Plato is contingently an animal, and so on for other singulars, therefore every man is contingently an animal’ [Ockham, 1974, p. 715]. In cases where the modality is *in sensu compositionis*, however, different rules apply:

this rule is not generally true [*vera*] ‘all the singulars are necessary, therefore the universal is necessary’. Similarly . . . this rule is not general ‘the universal is necessary therefore the singulars are necessary’.  
[Ockham, 1974, p. 717]

Another inference that is not valid (*non valet*) is ‘all the singulars are possible, therefore the universal is possible’:

For it does not follow ‘this is possible: this contingent proposition is true; and this is possible: that contingent proposition is true, and so for the other singulars; therefore this is possible: every contingent proposition is true’. [Ockham, 1974, p. 718]

It is clear that in these chapters Ockham was not concerned with the problems discussed in modern treatises on probability and induction. Although the subject matter was described as induction, the problems addressed are those of deductive logic, in particular the relations between universal propositions — or to be more accurate propositions involving universal quantification — and their associated singular propositions. When, for example, he wrote that ‘this rule is not valid, the singulars are contingent, therefore the universal is contingent’,<sup>12</sup> it is quite clear that he meant *all* the singulars, and not merely some of them. The problems involved in generalisation from a finite sample were not even raised, let alone answered: here at least Ockham was not engaged in that kind of enquiry.

### *Jean Buridan*

Ockham never wrote a commentary on either the *Prior* or the *Posterior Analytics*, but one fourteenth-century nominalist who did was Jean Buridan (c.1300–1358). Buridan took it for granted that inductive arguments are invalid if only some of the singulars are considered: ‘an induction is not a good consequence [*bona consequentia*] unless all the singulars are enumerated in it. But we cannot enumerate all

<sup>11</sup>Modal propositions *in sensu divisionis* (or *in sensu diviso*) were those where the modal operator was applied to part of the proposition, not the whole; propositions *in sensu compositionis* (or *in sensu composito*) were those where the operator was applied to the whole proposition: see [Broadie, 1993, pp. 59–60]; on Ockham’s usage, see [Lagerlund, 2000, pp. 98–100].

<sup>12</sup>*ista regula non valet, singulares sunt contingentes, igitur universalis est contingens*, ch. 36, [Ockham 1974, p. 720].

of them because they are infinitely many.’ [Biard, 2001, p. 92]. We do nevertheless draw general conclusion from finite samples:

For when you have often seen rhubarb purge bile and have memories of this, and have never found a counterexample in the many different circumstances you have considered, then the intellect, not as a necessary consequence, but only from its natural inclination to the truth, assents to the universal principle and understands it as if it were an evident principle based on an induction such as ‘this rhubarb purged bile, and that [rhubarb]’, and so on for many others, which have been sensed and held in memory. Then the intellect supplies the little clause [*clausulam*] ‘and so on for the [other] singulars’, because it has never witnessed a counterexample . . . nor is there any reason or dissimilarity apparent why there should be a counterexample. [Biard, 2001, p. 93]

Parts of this may remind a modern reader of Hume’s account of the operation of the mind, but there is one crucial difference: Buridan’s ‘inclinations’ are inclinations *to the truth*, not mere habits grounded on the association of ideas. In the background there is the unquestioned assumption — notoriously absent in Hume — that God has equipped us with faculties that, when not mis-used, will lead us to truth rather than error.

### 3 THE RENAISSANCE

#### *3.1 The revival of rhetoric*

At the risk of some simplification, it seems fair to say that the Renaissance saw a rise in the status of rhetoric, and a fall in the status of logic, or at least formal logic, though the process was far from uniform or complete. Aristotle continued generally to be treated with respect, even by those who did not think of themselves as Aristotelians, but the refinements of later medieval logic, with its intricate subtleties and (to the humanists) barbarous grammar and terminology, were quite another matter.

Hostility towards formal logic can be traced back at least as far as Petrarch, but the first sustained attack was at the hands of Lorenzo Valla (1407–1457). The earliest textbook in the new style was the *De Inventione Dialectica* of Rudolph Agricola (1444–1485), first published in 1515 and reprinted sufficiently often thereafter for it to have been described as ‘the first humanist work in logic to become a best seller’ [Monfasani, 1990, p. 181]. Similar criticisms of scholastic logic were made by Juan Luis Vives (1492–1540), who had studied logic in the University of Paris as an undergraduate, and had not enjoyed the experience [Broadie, 1993, pp. 192–206]; his *In Pseudodialecticos* was first published in 1520 [Vives, 1979].

Valla’s opposition to traditional logic was deeper than that of his successors, in that he disliked not merely late medieval subtleties, but formal logic as such [Mack, 1993, pp. 83–4]. The rules of sound reasoning, like the rules of good writing,

were to be drawn *ad consuetudinem eruditorum atque elegantium* [Valla, 1982, p. 217], that is, from the actual Latin usage of the best writers of the best period. Logic therefore became merely one part — and a relatively unimportant one at that — of rhetoric. Valla explicitly indicated his dislike of, and dissent from, the Boethian description of induction as a progression from particulars to universals [Valla, 1982, p. 346]: for him it was the rhetorical argument from particulars to particulars that mattered.

Agricola preferred the term ‘enumeratio’ to ‘inductio’, even though both had been used by Cicero: ‘to me it seems that induction should be more rightly called enumeration, since Cicero called it an argument from the enumeration of all the parts’ [Agricola, 1992, p. 316]. Some of the examples given are inductions of the traditional kind, but some certainly are not, for example: ‘the wall is mine, the foundation is mine, the roof is mine, the rest of the parts are mine. Therefore the house is mine.’ [Agricola, 1992, p. 316]. This kind of argument seems to have become a recognised type of induction in the rhetorical tradition: in the early eighteenth century Vico’s *Institutiones Oratoriae* drew a distinction between two kinds of induction, *inductio partium* and *inductio similibium*. The former in turn had two sub-varieties, one involving an enumeration of all the species that made up a genus, the other an enumeration of all the parts that make up a totality, such as the limbs and organs of the human body [Vico, 1996, p. 90].

### 3.2 Zabarella

One of the most interesting sixteenth-century accounts of induction was by one of the professors at Padua, at that time the leading university in Italy, and arguably in Europe, and one where the study of logic continued to flourish [Grendler, 2002, pp. 250–253, 257–266]. Jacopo Zabarella (1533–1589) has been described by Charles Schmitt as ‘in the methodological matters . . . without a doubt the most acute and most influential of the Italian Renaissance Aristotelians’ [Schmitt, 1969, p. 82]. In chapter 4 of his short treatise *De Regressu*, he distinguished two kinds of induction: dialectical and demonstrative. Dialectical induction is used when the subject matter is mutable and contingent (*in materia mutabili et contingente*) and has no strength (*nil roboris habet*) unless all the particulars are considered without exception [Zabarella, 1608, col. 485D]. Demonstrative induction, by contrast, can be employed

in necessary [subject] matter, and in things which have an essential connection among themselves, and for that reason in it [demonstrative induction] not all the particulars are considered, for our mind having inspected certain of these at once grasps the essential connection [*statim essentialem connexum animadvertit*], and leaving aside the remainder of the singulars, at one infers [*colligit*] the universal: for it knows it to be necessary that things are thus with the remainder [Zabarella, 1608, col. 485D–E].



#### 4 THE SEVENTEENTH CENTURY AND EARLY EIGHTEENTH CENTURY

Many of the most original and creative philosophers of the seventeenth and early eighteenth century had little or nothing to say about induction. The word does not appear in either Spinoza's *Ethics* or Locke's *Essay*, and only once in passing in Berkeley's *Principles of Human Knowledge*, § 50. That Spinoza had nothing to say is perhaps not very surprising,<sup>13</sup> but the reason for Locke's virtual silence — the term was used once in *The Conduct of the Understanding*<sup>14</sup> — is less immediately obvious. Part of the explanation may be that he had no confidence that natural philosophy would ever become a science, and that his own experience had mainly been as a physician, reasoning about particular cases and using general rules only as fallible guides to practice.

##### 4.1 Bacon

Though Francis Bacon was the first thinker to invert the traditional priority and give induction precedence over deduction, it is potentially misleading to describe him as the founder of inductive logic. Bacon was not a logician either by temperament or doctrine, and it would be unhelpful to see him as a remote precursor of Carnap. His treatment of induction should be seen in the context of a massive but incomplete programme for the discovery of a new kind of scientific knowledge [Malherbe, 1996; Gaukroger, 2001, pp. 132–159].

Like Descartes a generation later, Bacon while still quite young became profoundly dissatisfied with all the many and various kinds of natural philosophy currently taught in the universities, but while Descartes was repelled by the uncertainty of this so-called knowledge, Bacon despised it for its uselessness — its utter failure to provide any grounding for practically effective techniques of controlling nature.

Bacon's disdain for traditional philosophy was made plain in 1605, in his first major publication, the *Advancement of Learning*. His low opinion of the logic taught by the schoolmen extended to their treatment of induction:

Secondly, the Induction which the *Logicians* speake of, and which seemeth familiar with *Plato*, whereby the *Principles of Sciences* may be pretended to be invented, and so the middle propositions by derivation from the Principles; their forme of Induction, I say is utterly vitious and incompetent.... For to conclude *uppon an Enumeration of particulars, without instance contradictorie* is no conclusion but a coniecture; for who can assure (in many subjects) uppon those particulars which appeare of a side, that there are not other on the contrarie side, which not? [Bacon, 2000a, pp. 109–110]

<sup>13</sup>The word does occur in ch. 11 of the *Tractatus Theologico-Politicus* [Spinoza, 2004, p. 158].

<sup>14</sup>Our observations 'may be establish'd into Rules fit to be rely'd on, when they are justify'd by a sufficient and wary Induction of Particulars' [Locke, 1706, p. 49].

What Bacon proposed to use instead of this vicious and incompetent form of induction is not explained, though he did promise the reader that ‘if God give me leave’ he would one day publish an account of his new method, which he called the Interpretation of Nature [Bacon, 2000a, p. 111].

The promise was eventually honoured in 1620 with the publication of Bacon’s most substantial philosophical work, the *Novum Organum*, designed as the second part, though the first to be published, of a massive — and unfinished — six-part project, the Great Instauration (*Instauratio Magna*). The title chosen for this second part made it clear that Bacon was making an open challenge to Aristotle. Aristotle’s logical works had become known collectively as the *Organon*, or tool, and the New Organon was intended not merely as a supplement, but as a replacement.

Bacon’s case against the traditional logic of the schools had two main strands. In the first place the old logic was concerned with *talk* rather than action:

For the ordinary logic professes to contrive and prepare helps and guards for the understanding, as mine does; and in this one point they agree. But mine differs from it in three points especially; viz., in the end aimed at; in the order of demonstration; and in the starting point of the inquiry.

For the end which this science of mine proposes is the invention not of arguments but of arts; not of things in accordance with principles, but of principles themselves; not of probable reasons, but of designations and directions for works. And as the intention is different, so accordingly is the effect; the effect of the one being to overcome an opponent in argument, of the other to command nature in action. [Bacon, 1857–74, IV, pp. 23–24]

Training in traditional logic encouraged the wrong kind of mental skills: it placed a premium on intellectual subtlety, but ‘the subtlety of nature is far greater than the subtlety of the senses and understanding’ (*Novum Organum*, I. 10). Facility with words and agility in debate are not what is required when one is trying to penetrate the workings of nature.

Secondly, by concentrating on the forms of argument, syllogistic logic draws attention away from defects in their matter, which are far more dangerous:

The syllogism consists of propositions, propositions consist of words, words are symbols of notions. Therefore if the notions themselves (which is the root of the matter) are confused and over-hastily abstracted from the facts, there can be no firmness in the superstructure. Our only hope therefore lies in a true induction. [*Novum Organum*, I. 14]

This is the first occasion in which induction was mentioned in the *Novum Organum* (as distinct from the parts of the *Instauratio Magna* that preceded it), but there is no subsequent explanation of how induction could contribute to the rectification of

defective concepts. One thing that is apparent, however, is that Bacon's approach was quite different to Descartes': there was no suggestion that the establishment of a set of clear and distinct ideas either could or should precede the investigations undertaken with their help. The improvement of concepts and the growth of knowledge had to take place together, by slow increments.

Despite these harsh remarks, Bacon did not reject syllogistic reasoning entirely, but he restricted its use to areas of human life where 'popular', superficial concepts are employed:

Although therefore I leave to the syllogism and these famous and boasted modes of demonstration their jurisdiction over popular arts and such as are matter of opinion (in which department I leave all as it is), yet in dealing with the nature of things I use induction throughout ... [Bacon, 1857–74, IV, p. 24]<sup>15</sup>

Bacon was the opposite of an 'ordinary language' philosopher: he had no belief whatever that the concepts embedded since time immemorial in common speech would prove to be the ones needed in a reformed natural philosophy — indeed quite the contrary. One of his fundamental objections to Aristotle was that Aristotle had taken as his starting point popular notions and merely ordered and systematised them, instead of replacing them by something better.

Bacon had no liking for neologisms, and whenever possible preferred 'to retain the ancient terms, though I sometimes alter the uses and definitions, according to the Moderate proceeding in Civill government' [Bacon, 2000a, p. 81]. But though he was prepared to retain the traditional vocabulary, the kind of induction he was planning to use would very unlike anything described by his predecessors:

In establishing axioms, another form of induction must be devised than has hitherto been employed, and it must be used for proving and discovering not first principles (as they are called) only, but also the lesser axioms, and the middle, and indeed all. For the induction which proceeds by simple enumeration is childish; its conclusions are precarious and exposed to peril from a contradictory instance; and it generally decides on too small a number of facts, and on those only which are at hand. [*Novum Organum*, I. 105]<sup>16</sup>

The fallibility of induction by simple enumeration could hardly be more clearly expressed. Bacon had no intention of retaining it and merely adding safeguards that would make its use less risky and any conclusions reached more probable. He wanted it to be discarded in favour of something entirely different:

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<sup>15</sup>See also the letter of 30 June 1622 to Fr. Redemptus Baranzan [Bacon, 1857–74, XIV, p. 375].

<sup>16</sup>Axioms here are not the axioms of modern mathematics and logic, but rather important general principles; the term comes from Stoic logic [Frede, 1974, pp. 32–37; Kneale and Kneale, 1962, pp. 145–147].

But the induction which is to be available for the discovery and demonstration of sciences and arts, must analyse nature by proper rejections and exclusions; and then, after a sufficient number of negatives, come to a conclusion on the affirmative instances. . . . But in order to furnish this induction or demonstration well and duly for its work, very many things are to be provided which no mortal has yet thought of; insomuch that greater labour will have to be spent in it than has hitherto been spent on the syllogism. [*Novum Organum*, I. 105]

The last part of this was a warning that Bacon's own account of this new kind of induction would (at this stage) be far from complete. He never supposed that his method could be described in detail, prior to its employment in actual investigations. The specimen given in the *Novum Organum* of an enquiry made using the new kind of induction was explicitly described as a First Vintage, or provisional interpretation (*interpretatio inchoata*, II. 20); a full account would have to wait until the final part of the *Instauratio Magna*, the *Scientia Activa*, which was never written, or indeed even begun. One thing that was clear from the start, however, is that it would be a form of eliminative induction, relying on 'rejections and exclusions'. A great mass of merely confirming instances, however large, is never enough.

Bacon's own preliminary account of his method is given in Book II of the *Novum Organum*. There are three stages: the compilation of a 'natural and experimental history' of the nature under investigation, the ordering of this in tables, and finally induction. While the first two of these are described in considerable detail (*Novum Organum*, II. 10–14), the account of induction itself is strikingly brief:

We must make, therefore, a complete solution and separation of nature, not indeed by fire, but by the mind, which is a kind of divine fire. The first work therefore of true induction (as far as regards the discovery of Forms) is the rejection or exclusion of the several natures which are not found in some instance where the given nature is present, or are found in some instance where the given nature is absent, or are found to increase in some instance when the given nature decreases, or to decrease when the given nature increases. [*Novum Organum*, II. 16]

Bacon's theory of forms is notoriously obscure — they are certainly not the substantial forms of the Aristotelians — but it is clear that, whatever they might be in ontological terms, they are the causes of the (phenomenal) natures [Pérez-Ramos, 1988, pp. 65–132]. The form of heat is something which is present in all hot bodies, absent from all cold bodies, and which varies in intensity according to the degree of heat found in a body.

The conclusion of the process of induction was described in a vivid (but opaque) metaphor taken from contemporary chemistry: 'after the rejection and exclusion has been duly made, there will remain at the bottom, all light opinions vanishing into smoke, a Form affirmative, solid, and true and well defined' (*Novum Organum*, II. 16), like a puddle of gold at the bottom of an alchemist's crucible. Bacon's own

comment on this is entirely apposite: ‘this is quickly said; but the way to come at it is winding and intricate.’

Bacon’s confidence that his method of eliminative induction would produce certain knowledge rested on several presuppositions, of which the most important is what Keynes subsequently termed a Principle of Limited Variety. Though the world as we experience it appears unendingly varied, all this complexity arises from the combination of a finite — indeed quite small — number of simple natures. There is an alphabet of nature,<sup>17</sup> the contents of which cannot be guessed or discovered by speculation, but which will start to be revealed once the correct inductive procedures are employed. Bacon made no attempt to give an a priori justification of this, and there is no reason to suppose that he would have regarded any such justification as either possible or necessary. As always, validation would be retrospective — by having supplied those who employed the method correctly with power over nature.

## 4.2 *Descartes*

Induction played no significant role in Descartes’ mature philosophy, but there are some remarks on it in the early and unfinished *Regulae ad Directionem Ingenii* (c.1619–c.1628). Whether Descartes had read any of Bacon’s works at this stage in his life is not known — he certainly became familiar with Bacon’s thought subsequently [Clarke, 2006, p. 104] — but his account in the *Regulae* appears to have owed nothing whatever to the *Novum Organum*.

In the *Regulae* the most certain kind of knowledge comes from intuition, a direct apprehension of the mind unmediated by any other intellectual operations. Deduction is needed because some chains of reasoning are too complex to be grasped by a single act of thought: we can grasp intuitively the link between each element in the chain and its predecessor, but not all the links between the elements at once.

Induction<sup>18</sup> was dealt with more briefly, Rule VII stating that

In order to make our knowledge complete, every single thing relating to our undertaking must be surveyed in a continuous and wholly uninterrupted sweep of thought, and be included in a sufficient and well ordered enumeration [*sufficienti et ordinata enumeratione*]. [Descartes, 1995, I, p. 25]

It would seem that for Descartes the words ‘inductio’ and ‘enumeratio’ were merely alternative names for the same thing; their equivalence is suggested by phrases such as ‘enumeratio, sive inductio’ and ‘enumerationem sive inductionem’ in the passages quoted below [Descartes, 1908, pp. 388, 389], [Marion, 1993, p. 103].

<sup>17</sup>On this, and Bacon’s work on an *Abececlarium Naturae*, see [Bacon, 2000b, pp. xxix–xl, 305].

<sup>18</sup>The word occurs three times in Rule VII [Descartes, 1908, pp. 388, 389, 390] and once in Rule XI (p. 408). There is one place in Rule III (p. 368) where ‘inductio’ appears in the first edition of 1701, but this may be a transcriber’s or printer’s error for ‘deductio’; for a discussion of the problem, see [Descartes, 1977, pp. 117–119].

The function of a sufficient enumeration is given in the explication of Rule VII:

We maintain furthermore that enumeration is required for the completion of our knowledge [*ad scientiae complementum*]. The other Rules do indeed help us resolve most questions, but it is only with the aid of enumeration that we are able to make a true and certain judgement about whatever we apply our minds to. By means of enumeration nothing will wholly escape us and we shall be seen to have some knowledge on every question.

In this context enumeration, or induction, consists in a thorough investigation of all the points relating to the problem at hand, an investigation which is so careful and accurate that we may conclude with manifest certainty that we have not inadvertently overlooked anything. So even though the object of our enquiry eludes us, provided we have made an enumeration we shall be wiser at least to the extent that we shall perceive with certainty that it could not possibly be discovered by any method known to us. [Descartes, 1995, I, pp. 25–26]

If an enumeration is to lead to a negative conclusion that the knowledge of something lies entirely beyond the reach of the human mind, then it is essential that it should be ‘sufficient’:

We should note, moreover, that by ‘sufficient enumeration’ or ‘induction’ [*sufficientem enumerationem sive inductionem*] we just mean the kind of enumeration which renders the truth of our conclusions more certain than any other kind of proof [*aliud probandi genus*] (simple intuition excepted) allows. But when our knowledge of something is not reducible to simple intuition and we have cast off our syllogistic fetters, we are left with this one path, which we should stick to with complete confidence. [Descartes, 1995, I, p. 26]

This notion of a sufficient enumeration plays a crucial role in Descartes’ account, and it is unfortunate that his explication of it singularly fails to meet his own professed ideal of perfect clarity [Beck, 1952, p. 131]. It is not the same as completeness. If I wish to determine how many kinds of corporeal entities there are, I need to distinguish them from one another and make a complete enumeration of all the different kinds,

But if I wish to show in the same way that the rational soul is not corporeal, there is no need for the enumeration to be complete; it will be sufficient if I group all bodies together into several classes so as to demonstrate that the rational soul cannot be assigned to any of these. [Descartes, 1995, I, pp. 26–27]

The thought here appears to be that we do not need to make a complete list of all the different kinds of body: if we are merely attempting to establish a negative

thesis about what the rational soul is not, a division into several broad classes is enough.

One relatively straightforward example of an enumeration is given in Rule VIII. Someone attempting to investigate all the kinds of knowledge will have to begin by considering the pure intellect, since the knowledge of everything else depends on this; then ‘among what remains he will enumerate [*enumerabit*] whatever instruments of knowledge we possess in addition to the intellect; and there are only two of these, namely imagination and sense perception’ [Descartes, 1995, I, p. 30].

He will make a precise enumeration [*enumerabit exacte*] of all the paths to truth which are open to men, so that he may follow one which is reliable. There are not so many of these that he cannot immediately discover them all by means of a sufficient enumeration [*sufficientem enumerationem*]. . . [Descartes, 1995, I, p. 30]

Another example of an enumeration — here explicitly described as an induction — is more puzzling:

To give one last example, say I wish to show by enumeration that the area of a circle is greater than the area of any other geometrical figure whose perimeter is the same length as the circle’s. I need not review every geometrical figure. If I can demonstrate that this fact holds for some particular figures, I shall be entitled to conclude by induction that the same holds true in all the other cases as well. [Descartes, 1995, I, p. 27]

This does not appear to be an example of a complex proof composed of separate proofs of a finite set of more specific cases, as when a theorem about triangles in general is established by showing it to be true for acute, obtuse and right-angled triangles. There are clearly an infinite number of polygons with the same perimeter as a given circle but with smaller areas. How the argument is meant to proceed is not clear, but it is certainly not though an exhaustive case-by-case analysis.<sup>19</sup>

There is no further discussion of induction in the works that Descartes published. The *Regulae* was not printed until 1701, though copies circulated in manuscript and were read by Leibniz, and (possibly) by Locke.<sup>20</sup> The account of knowledge that Locke gave in book IV of the *Essay concerning Human Understanding* certainly has close parallels with the account in the *Regulae*, but there is no mention at all of induction.

### 4.3 Gassendi

Pierre Gassendi discussed induction in his *Institutio Logica* [Gassendi, 1981], first published in 1658 as parts of his posthumous *Opera Omnia*. Following in the

<sup>19</sup>The result is not elementary. A non-rigorous proof, first given by Zenodorus (2nd century BC?), is preserved in Book V of Pappus’ *Collections* [Cuomo, 2000, pp. 61–62].

<sup>20</sup>If Locke had seen a copy, it would have been between 1683 and 1689 when he was in exile in the Netherlands. There is no mention of this anywhere in his private papers.

tradition of the *Prior Analytics*, induction was treated as a kind of syllogism, for example:

Every walking animal lives, every flying animal lives, and also every swimming animal, every creeping animal, every plant-like animal; therefore every animal lives. [Gassendi, 1981, p. 53]

In such an induction there is a concealed premise:

Every animal is either walking, or flying, or swimming, or creeping, or plant-like.

Without this, the inference would have no force (*consequutionis vis nulla foret*), since if there were another kind of animal in addition to these, a false conclusion could emerge. If an induction is to be valid (*legitima*) it has to be based on an enumeration of all the relevant species or parts, and as Gassendi commented, such an enumeration is usually difficult if not impossible to achieve [Gassendi, 1981, p. 54].

The same account appeared with only minor changes in the French *Abregé de la philosophie de Gassendi*, published by Gassendi's disciple, François Bernier [Bernier, 1684, I, pp. 132–133].

#### 4.4 *Arnauld and Nicole*

The work most strongly influenced by Descartes' as yet unpublished *Regulae* was *La Logique ou l'art de penser*, published in 1662 by Antoine Arnauld and Pierre Nicole. Induction is introduced in traditional and broadly neutral terms:

Induction occurs whenever an examination of several particular things leads us to knowledge of a general truth. Thus when we experience several seas in which the water is salty, and several rivers in which the water is fresh, we infer that in general sea water is salty and river water is fresh. [Arnauld and Nicole, 1996, p. 202]

Induction is described as the beginning of all knowledge, because singular things are presented to us before universals. This sounds thoroughly Aristotelian, but the resemblance is only superficial. Though I might never have started to think about the nature of triangles if I had not seen an individual example, 'it is not the particular examination of all triangles which allows me to draw the general and certain conclusion about all of them . . . but the mere consideration of what is contained in the idea of the triangle which I find in my mind' [Arnauld and Nicole, 1996, p. 202]. The same is true of the very general axioms that have application in fields quite remote from geometry, for example the principle that a whole is greater than its part, the ninth and last of Euclid's Common Notions. According to certain philosophers — un-named, but presumably Gassendi and his followers — we know this only because ever since our infancy we have observed that a man is larger than his head, a house larger than a room, a forest larger than a tree, and

so on. Arnauld and Nicole replied that ‘if we were sure of this truth . . . only from the various observations we had made since childhood, we would be sure only of its probability [*nous n’en serions probablement assurés*], since induction is a certain means of knowing something only when the induction is complete’ [Arnauld and Nicole, 1996, p. 247].

It is striking that several of Arnauld and Nicole’s examples of over-confident reliance on inherently fallible inductive arguments are taken from recent developments in the physical sciences. Natural philosophers had long believed that a piston could not be drawn out of a perfectly sealed syringe and that a suction pump could lift water from any depth, and they supposed these alleged truths to be founded on a ‘a very certain induction based on an infinity of experiments [*expériences*]’ [Arnauld and Nicole, 1996, p. 203, translation modified]. Again, it was assumed that if water was contained in a curved vessel (e.g. a U-tube) of which one arm was wider than the other, the level in the two arms would be equal; experiment had shown that this was not true when one arm was very narrow, allowing capillary attraction to become significant [Arnauld and Nicole, 1996, p. 247]. It would probably be going to far to say that new discoveries in the natural sciences were the main force fuelling inductive scepticism, but they do seem to have played a part in reducing confidence in the age-old experiential data on which Aristotelian science had been based [Dear, 1995].

#### 4.5 *Hobbes and Wallis*

Despite Hobbes’s strong and unswerving commitment to nominalism, inductive reasoning did not play a large role in his philosophy, and he had little to say about it. In the mid-1650s he became involved in a series of acrimonious arguments with the mathematician John Wallis [Jesseph, 1999], part of which touched on Wallis’s use of inductive arguments. Hobbes thought that induction had no place in mathematics, or at least in mathematical demonstration:

*The most simple way (say you) of finding this and some other Problemes, is to do the thing it self a little way, and to observe and compare the appearing Proportions, and then by Induction, to conclude it universallly. Egregious Logicians and Geometricians, that think an Induction without a Numeration of all the particulars sufficient to infer a Conclusion universall, and fit to be received for a Geometricall Demonstration!* [Hobbes, 1656, p. 46]

Hobbes clearly thought that there were only two kinds of induction, one founded on a complete enumeration of all the particulars, which could lead to certainty, and the other founded on a partial enumeration, which could not.

The remarks by Wallis that Hobbes had found so objectionable were in the *Arithmetica Infinitorum* of 1656, but his fullest treatment of the issue is to be found in his much later *Treatise of Algebra* [Wallis, 1685], where he was responding to the criticisms of a very much more capable mathematician than Hobbes, Pierre

Fermat [Stedall, 2004, pp. xxvi–xxvii]. Wallis insisted that inductive arguments have a legitimate role in mathematics:

As to the thing itself, I look upon *Induction* as a very good Method of *Investigation*; as that which doth very often lead us to the easy discovery of a General Rule; or is at least a good preparative to such an one. And where the Result of such Inquiry affords to the view, an obvious discovery; it needs not (though it may be capable of it,) any further Demonstration. And so it is, when we find the Result of such Inquiry, to put us into a regular orderly Progression (of what nature soever,) which is observable to proceed according to one and the same general Process; and where there is no ground of suspicion why it should fail, or of any case which might happen to alter the course of such Process. [Wallis, 1685, p. 306]

The example Wallis gave of this was the expansion of the binomial  $(a + e)^n$ :  $(a + e)^2 = a^2 + 2ae + e^2$ ,  $(a + e)^3 = a^3 + 3a^2e + 3ae^2 + e^3$ ,  $(a + e)^4 = a^4 + 4a^3e + 6a^2e^2 + 4ae^3 + e^4$ , and so on. The coefficients in each case can be found by repeated multiplication, but more easily by using the diagram now known as Pascal's triangle, in which each element is the sum of the two diagonally above it in the row above:

$$\begin{array}{cccccccc}
 & & & & & & & 1 \\
 & & & & & & 1 & & 1 \\
 & & & & & 1 & & 2 & & 1 \\
 & & & 1 & & 3 & & 3 & & 1 \\
 & & 1 & & 4 & & 6 & & 4 & & 1 \\
 1 & & & 5 & & 10 & & 10 & & 5 & & 1
 \end{array}$$

The general result, that this procedure can be used for any power, is described as being established by induction. Wallis remarked:

But most Mathematicians that I have seen, after such Induction continued for some few Steps, and seeing no reason to disbelieve its proceeding in like manner for the rest, are satisfied (from such evidence,) to conclude universally, *and so in like manner for the consequent Powers*. And such Induction hath hitherto been thought (by such as do not list to be captious) a conclusive Argument. [Wallis, 1685, p. 308]

There is no indication here or anywhere else that the mathematical results reached by this kind of induction are merely probable, or in any way uncertain. The reason is clear: 'there is, in the nature of Number, a sufficient ground for such a sequel' [Wallis, 1685, p. 307]. Wallis was not using what has since become known as mathematical induction [Cajori, 1918]: his argument is much closer to the demonstrative induction of the later Aristotelians such as Zabarella, in which the examination of a few cases is enough to reveal the underlying regularity.

Wallis's account of induction in Book III, chapter 15 of his *Institutiones Logicae* [Wallis, 1687, pp. 167–172] is more traditional, and much of it is concerned with the reduction of perfect inductions to various figures of the syllogism. In imperfect inductions the conclusion is described as only conjectural, or probable, on the familiar grounds that it can be overturned by a single negative instance [Wallis, 1687, p. 170]. No attempt was made to estimate any degrees of probability. Wallis insisted several times that the weakness (*imbecillitas*) that characterised all imperfect inductions did not lie in their form, which was that of a syllogism, but in their matter:

For example, if someone argues *Teeth in the upper jaw are absent in all horned animals*; because it is thus in *the Ox, the Sheep, the Goat, nor is it otherwise* (as far as we know) *in the others*; Therefore (at least as far as we know) *in all*. This conclusion is not certain, but only probable [*verisimilis*]; not through a defect in the Syllogistic form, but through the uncertainty of the matter, or the truth of the premises. [Wallis, 1687, pp. 170–171]

In other words an inductive argument is not a fallible inference from reliably established premises such as ‘The ox has no teeth in the upper jaw’ and ‘The sheep has no teeth in the upper jaw’, but rather a deductive inference from premises like ‘The ox, the sheep and the goat have no teeth in the upper jaw, nor is it otherwise in the other horned animals’, all of which are uncertain and are provisionally accepted merely because no counter-examples are known to exist.

#### 4.6 Leibniz

Induction was not a central issue in Leibniz's philosophy, but given his omnivorous intellectual curiosity, it is not surprising either that he said something or that what he had to say is of considerable interest [Rescher, 1981; 2003; Westphal, 1989].

In the preface to his *New Essays on Human Understanding* Leibniz explained that one point of fundamental disagreement between him and Locke concerned the existence or non-existence of innate principles in the soul. This in turn raised the question of ‘whether all truths depend on experience, that is on induction and instances, or if some of them have some other foundation’. Leibniz chose the second answer: the senses ‘never give us anything but instances, that is particular or singular truths. But however many instances confirm a general truth, they do not suffice to establish its universal necessity; for it does not follow that what has happened will always happen in the same way.’ [Leibniz, 1981, p. 49]. Our knowledge of truths of reason, such as those of arithmetic and geometry, is not based on induction at all.

Not all truths are, however, truths of reason, and the other kind of truths — truths of fact — need to be discovered in a different way, at least by human beings. As Leibniz remarked in a paper ‘On the souls of men and beasts’, written around 1710:

there are in the world two totally different sorts of inferences, empirical and rational. Empirical inferences are common to us as well as to beasts, and consist in the fact that when sensing things that have a number of times been experienced to be connected we expect them to be connected again. Thus dogs that have been beaten a number of times when they have done something displeasing expect a beating again if they do the same thing, and therefore they avoid doing it; this they have in common with infants. [Leibniz, 2006, p. 66]

Beasts and infants are not alone in making such inferences: so too do human beings. As he noted in § 28 of the *Monadology*,

Men act like beasts insofar as the sequences of their perceptions are based only on the principle of memory, like empirical physicians who have a simple practice without theory. We are all mere empirics in three-fourths of our actions. For example, when we expect daylight tomorrow, we act as empirics, because this has always happened up to the present. Only the astronomer concludes it by reason. [Leibniz, 1969, p. 645, translation modified]<sup>21</sup>

There is however one difference between beasts and mere empirics: ‘beasts (as far as we can tell) are not aware of the universality of propositions . . . And although empirics are sometimes led by inductions to true universal propositions, nevertheless it only happens by accident, not by the force of consequence.’ [Leibniz, 2006, p. 67]. Human beings when relying purely on experience make generalisations that are often wrong, but beasts seem not to generalise at all.

It would seem from this that there are three kinds of reasoning (using this word in a large sense): (1) inferences from particulars to other particulars, the kind of reasoning that earlier philosophers had called *paradeigma* or example; (2) inductive generalisation proper; and (3) deduction. Something of this kind seems to be indicated in a note he made on the back of a draft letter dated May 1693, where a distinction is made between three grades of confirmation (*firmitas*):

logical certainty, physical certainty, which is only logical probability, and physical probability. The first example [is] in propositions of eternal truth, the second in propositions which are known to be true by induction, as that every man is a biped, for sometimes some are born with one foot or none; the third that the south wind brings rain, which is usually true but not infrequently false. [Couturat, 1961, p. 232]

Physical certainty is identified with moral certainty in *New Essays* IV. vi. 13 [Leibniz 1981, p. 406]. The implication is that the conclusions reached by inductive inferences can at least sometimes be morally certain.

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<sup>21</sup>The parallel between beasts and empirical physicians is one that Leibniz drew several times: *Principles of Nature and Grace*, § 5 [Leibniz, 1969, p. 638], *New Essays*, preface, [Leibniz, 1981, p. 50].

An indication of how such certainty can be obtained is provided by one of the earliest expositions of Leibniz's views on induction. The *Dissertatio de Stilo Philosophico Nizzolii* was a preface written in 1670 for a new edition of Mario Nizzoli's *De Veris Principiis et Vera Ratione Philosophandi contra Pseudophilosophos*, first published in 1553. Nizzoli (1488–1567) was an idiosyncratic thinker who has been described as a Ciceronian Ockhamist, and for Leibniz, his fundamental error is his nominalism — his denial of the existence of real universals:

If universals were nothing but collections of individuals, it would follow that we could attain no knowledge through demonstration . . . but only through collecting individuals or by induction.<sup>22</sup> But on this basis knowledge would straightway be made impossible, and the skeptics would be victorious. For perfectly universal propositions can never be established on this basis because you are never certain that all individuals have been considered. You must always stop at the proposition that all the cases which I have experienced are so. But . . . it will always remain possible that countless other cases which you have not examined are different. [Leibniz, 1969, p. 129]

Leibniz admitted that we believe confidently that fire burns, and that we will ourselves be burned if we place our hand in one, but this kind of moral certainty does not depend on induction alone and is reached only with the assistance of other universal propositions:

1. if the cause is the same or similar in all cases, the effect will be the same or similar in all;
2. the existence of a thing which is not sensed is not assumed; and, finally,
3. whatever is not assumed, is to be disregarded in practice until it is proved.

[Leibniz, 1969, p. 129].

The second and third of these are methodological principles, similar though not identical to Ockham's Razor. The first is a more carefully worded version of Hume's principle that 'like causes always produce like effects'.<sup>23</sup>

Without the aid of these helping propositions (*adminicula*), as Leibniz called them, not even moral certainty would be possible. Our knowledge of the *adminicula* cannot therefore be grounded on induction: 'For if these helping propositions, too, were derived from induction, they would need new helping propositions, and so on to infinity' [Leibniz, 1969, p. 130]. Leibniz's language here was Baconian,<sup>24</sup> but his thought manifestly is not: it is much closer to Hume.

<sup>22</sup>As the Latin (*collectionem singularium, seu inductionem*) makes clear, this is one process named in two ways, not two distinct processes [Leibniz, 1840, p. 70].

<sup>23</sup>The sixth of Hume's rules by which to judge of causes and effects, *Treatise of Human Nature*, I. iii. 15.

<sup>24</sup>The *Adminicula inductionis* were announced as a topic of future discussion in *Novum Organum*, II. 21, but never described in detail.

There is an illuminating comparison to be made between this kind of sophisticated induction, buttressed by the *adminicula*, and the demonstrative induction described by Zabarella. In demonstrative induction the conclusion can be made certain to us because the intellect grasps the universal nature on which the truth of the universal proposition is grounded. In Leibniz the help is provided by principles of a much higher degree of generality, such as the Law of Continuity [Leibniz, 1969, pp. 351–352], and ultimately the Principle of Sufficient Reason. In the words of Foucher de Careil:

Thus Leibniz has seen that in order to be introduced into science, induction needs the help of certain universal propositions that in no way depend on it. And since there can be no obstacle to the complete and systematic unity of science except the diversity of the facts of experience, he saw that the law of continuity, which is the link between the universal and the particular, and which unites them in science, is the true basis of induction . . . without it induction is sterile, with it, it generates moral certainty. [Leibniz, 1857, p. 422]

It is the regularity of nature — i.e. the fact that it is law-governed — that makes properly conducted inductive inferences safe.

## 5 CONCLUSION

The story told in the pages above has been an episodic and fragmentary one, with remarks about induction extracted from the writings of authors who were almost always concerned primarily with other matters, and for whom inductive reasoning was a matter of relatively minor importance. The one significant exception was Bacon, and even his treatment of his new method of eliminative induction was remarkably brief, given its pivotal role in his programme.

Even in the modern world a philosopher is not required to deal with induction at any length — or indeed at all — if they wish to be considered as a candidate for greatness. Given the direction of his interests, no one would have expected Nietzsche, for instance, to have focused his considerable talents on the problem, and the same is true of a large number of his predecessors. What is striking is not that many philosophers chose to concentrate on other matters, but that virtually everyone did.

The notion that there is a general and far-reaching ‘problem of induction’ is relatively recent. One of the earliest and most influential uses of the phrase was in J. S. Mill’s *System of Logic*, III. iii. 3, where the discussion of induction concluded with the following peroration:

Why is a single instance, in some cases, sufficient for a complete induction, while in others, myriads of concurring instances, without a single exception known or presumed, go such a very little way towards establishing an universal proposition? Whoever can answer this question

knows more of the philosophy of logic than the wisest of the ancients,  
and has solved the problem of induction. [Mill, 1973–4, p. 314]

Whether anyone has subsequently succeeded in solving — or dissolving — the problem may be doubted, though confident (and sometimes absurd) claims have continued to be made. What does seem clear is that no one before the nineteenth century saw induction as posing a single, general problem, still less regarded a failure to solve it as being, in C. D. Broad's often-quoted words, 'the scandal of Philosophy' [Broad, 1926, p. 67].

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